Computationally Efficient Calibration Algorithm for Three-Axis Accelerometer and Magnetometer

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***Abstract*—In this work we propose an algorithm for calibration of both 3-axis accelerometers and 3-axis magnetometers based on a computationally efficient modification of a popular calibration method. The proposed algorithm achieves fast convergence without requiring any external piece of equipment like a turntable. The evaluation of the proposed algorithm is done using experimental data.**

***Index Terms*—accelerometer, magnetometer, calibration, hard-iron, soft-iron**

I. Introduction

Accelerometers in combination with magnetometers are used in many applications including navigation and heading estimation. For low-cost applications, Micro-Electro-Mechanical (MEMS) accelerometers are most commonly used due to their cost effectiveness and small size. However, MEMS-based accelerometers lack in measurement accuracy mostly due to their manufacturing imperfections. Thus, when accuracy is needed, a calibration procedure is required.

In order to calibrate a 3-axis accelerometer, most authors use the fact that the magnitude of gravity is constant. Based on that, and assuming a number of accelerometer measurements while the sensor is still, they most commonly formulate a maximum likelihood estimation problem [1] or a linear least-squares problem [2] to estimate the calibration parameters.

Magnetometers also require a calibration procedure to compensate for their measurement errors. In case of magnetic sensors, the dominant part of their measurement error is caused by nearby materials which distort the measured magnetic field. More specifically, hard-iron distortion is created by magnetic materials attached to the magnetometer's reference frame and causes a bias to the measurements. Soft-iron distortion is caused by materials attached to the magnetometer's reference frame which influence the measured magnetic field but don't generate a magnetic field.

Similarly to the accelerometer case, the constant magnitude of the magnetic field is usually exploited for magnetometer calibration. Authors in [3] and [4] solve a maximum likelihood estimation problem to estimate the calibration parameters. In [5] a Kalman filter is applied to a properly formulated state estimation problem for magnetometer calibration.

In many applications, accelerometer and magnetometer calibration are implemented as part of an embedded system. Thus, in such applications, the calibration algorithm must be computationally efficient in order to be implemented in microcontrollers or Field-Programmable Gate Arrays (FPGAs) with limited hardware resources. In this work we propose a modification of a popular calibration approach [4] [6], resulting in a quadratic optimization problem, for both accelerometer and magnetometer calibration. The proposed algorithm excels in computational efficiency and provides fast convergence requiring no external piece of equipment. The proposed algorithm is evaluated using experimental data.

This paper is organized as follows. In section II accelerometer's and magnetometer's measurement models are presented. The proposed calibration algorithm is described in Section III. Finally, evaluation of the proposed algorithm using experimental data and conclusions are presented in sections IV and V.

II. Measurement Models

For accelerometer and magnetometer calibration, a mathematical model that relates the sensors' measurements with the true values of specific force and magnetic field respectively is required.

A. Accelerometer Measurement Model

The measurement of an accelerometer is modeled as [7] [8]

$$y_a = f + T_{sf} f + T_{cc} f + h_a + \varepsilon, \tag{1}$$

where

y_a : 3×1 measurement vector
f : 3×1 true specific force vector
T_{sf} : 3×3 diagonal matrix representing the scale-factor error
T_{cc} : 3×3 matrix representing the cross-coupling error
h_a : 3×1 accelerometer's bias vector
ε : random error

Defining $T_a \triangleq I_3 + T_{sf} + T_{cc}$, where I_3 is the 3×3 identity matrix, (1) can be written as

$$y_a = T_a f + h_a + \varepsilon \tag{2}$$

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B. Magnetometer Measurement Model

The measurement of a magnetometer is modeled as [6], [9], [4], [10]

$$y_m = T_{sf} T_{cc} \left(T_{si} m + h_{hi}\right) + h_b + \varepsilon \tag{3}$$

where

$y_m : 3 \times 1$ measurement vector
$m : 3 \times 1$ true magnetic field vector
$T_{sf} : 3 \times 3$ diagonal matrix representing the scale-factor error
$T_{cc} : 3 \times 3$ matrix representing the cross-coupling error
$T_{si} : 3 \times 3$ matrix representing the soft-iron distortion
$h_b : 3 \times 1$ magnetometer's bias vector
$h_{hi} : 3 \times 1$ bias vector due to hard-iron distortion
$\varepsilon :$ random error

Setting $T_m \triangleq T_{sf} T_{cc} T_{si}$ and $h_m \triangleq T_{sf} T_{cc} h_{si} + h_b$, the magnetometer's measurement model becomes

$$y_m = T_m m + h_m + \varepsilon \tag{4}$$

As seen, (2) and (4) share the same form, so the general model (5) can be used for both sensors' calibration.

$$y = Tn + h + \varepsilon \tag{5}$$

III. Calibration algorithm

A popular calibration approach ([4], [6]) uses the fact that the measured magnitude of the specific force or magnetic field should be constant. Assuming N measurements and using (5), the calibration algorithm is formulated as the error minimization problem in (6).

$$\begin{aligned} \text{minimize} \quad & \sum_{k=1}^{N} \|y_k - Tn_k - h\|^2 \\ \text{subject to} \quad & \|n_k\| = 1, \ k = 1, 2, ..., N \end{aligned} \tag{6}$$

For both accelerometer and magnetometer, without loss of generality, we assume the magnitude of the specific force and magnetic field respectively is one. Note that both T and n_k in the product Tn_k are unknowns and therefore (6) is a quartic optimization problem. A popular approach to solving (6) is via the penalty function method, i.e., forming

$$J_0(x) = \sum_{k=1}^{N} \left\{ \|y_k - Tn_k - h\|^2 + \lambda \left(\|n_k\|^2 - 1\right)^2 \right\} \tag{7}$$

where $\lambda > 0$ and

$$x = \begin{bmatrix} \text{vec}(T)^T & h^T & n_1^T & \dots & n_N^T \end{bmatrix}^T$$

and minimizing it using the gradient descent method. The gradient vector of the penalty function $J_0(x)$ is

$$\nabla J_0(x) = \left[= \frac{\partial J_0(x)}{\partial vec(T)}^T \ \frac{\partial J_0(x)}{\partial h}^T \ \frac{\partial J_0(x)}{\partial n_1}^T \ \cdots \ \frac{\partial J_0(x)}{\partial n_N}^T \right]^T \tag{8}$$

Defining $u_k = y_k - h$, we have

$$\frac{\partial J_0(x)}{\partial vec(T)} = 2 \sum_{k=1}^{N} \left\{ n_k \otimes (Tn_k) - (n_k^T \otimes u_k^T)^T \right\}$$

$$\frac{\partial J_0(x)}{\partial h} = 2 \sum_{k=1}^{N} \{h - y_k + Tnk\}$$

$$\frac{\partial J_0(x)}{\partial n_l} = 2 \sum_{k=1}^{N} \{2n_k(n_k^T n_k - 1) - T^T u_k + T^T T n_k\}$$

where $\otimes$ denotes the Kronecker's product [11].

As already mentioned, optimization problem (6) is quartic because both T and n_k are unknowns and the penalty function involves the square of their product. This results in slow convergence as shown in Section IV. To overcome this and achieve a computationally efficient algorithm we introduce the following transformation which converts (6) into a quadratic problem. Specifically, we multiply (5) by $H = T^{-1}$ to derive

$$Hy = n + v + \tilde{\varepsilon}, \tag{9}$$

where $H \triangleq T^{-1}$, $v \triangleq T^{-1}h$ and $\tilde{\varepsilon} = T^{-1}\varepsilon$. Then, minimization of the square error $\tilde{\varepsilon}$ leads to the quadratic optimization problem (10).

$$\begin{aligned} \text{minimize} \quad & \sum_{k=1}^{N} \|Hy_k - n_k - v\|^2 \\ \text{subject to} \quad & \|n_k\| = 1, \ k = 1, 2, ..., N \end{aligned} \tag{10}$$

Note that $\tilde{\varepsilon}$ is a legitimate definition of the error as it adds directly to the specific force. We write the penalty function corresponding to (10)

$$J(x) = \sum_{k=1}^{N} \left\{ \|Hy_k - n_k - v\|^2 + \lambda \left(\|n_k\|^2 - 1\right)^2 \right\} \tag{11}$$

where $\lambda > 0$ and

$$x = \begin{bmatrix} \text{vec}(H)^T & v^T & n_1^T & \dots & n_N^T \end{bmatrix}^T$$

The gradient vector of the penalty function (11) is

$$\nabla J(x) = \left[\frac{\partial J(x)}{\partial \text{vec(H)}}^T \ \frac{\partial J(x)}{\partial v}^T \ \frac{\partial J(x)}{\partial n_1}^T \ \cdots \ \frac{\partial J(x)}{\partial n_N}^T \right]^T \tag{12}$$

where

$$\frac{\partial J(x)}{\partial \text{vec}(H)} = 2 \sum_{k=1}^{N} \left[y_k \otimes (Hy_k - n_k - v) \right]$$

$$\frac{\partial J(x)}{\partial v} = 2 \sum_{k=1}^{N} \left[-Hy_k + v + n_k \right]$$

$$\frac{\partial J(x)}{\partial n_k}\Big|_{k=1,2,\dots,N} = -2(Hy_k + n_k - v) + 4\lambda n_k \|n_k\|^2$$

Optimization problem (10) can be solved using the gradient descent method. In order for the gradient descent algorithm to converge, a good initial estimate of the unknown vector x is required.

For the accelerometer, under the reasonable assumption of small scale-factor and cross-coupling errors, an initial estimate of H is the 3×3 identity matrix (I_3). In a similar way, bias vector v is initialized as the 3×1 zero vector ($0_{3\times1}$).

For the magnetometer, deriving an initial estimate of the unknown vector x is not trivial due to the hard-iron and soft-iron distortions. To this end, the authors in [4] and [9] use a linear least-squares problem to find an initial estimate of x.

Finally, for better convergence, the backtracking line-search method [12] is incorporated in the gradient descent method.

The proposed algorithm is summarized in Algorithm 1.

Algorithm 1: Calibration Steps

Step 1: Initialize H and v

Step 2: Initialize $n_k = y_k, k = 1, 2, \ldots, N$,

Step 3: Initialize λ, t, a, b

Step 4: Determine the descent direction:
Δx=-∇J(x)

Step 5: Choose step size:
while $g(x + t\Delta x) > J(x) + at\nabla J(x)^T \Delta x$
t := βt

Step 6: Update $x = x + \Delta x$

Step 7: Calculate $J(x)$

Step 8: Repeat steps 4-7 until $J(x)$ is sufficiently small

IV. Experimental Results

The proposed algorithm is evaluated using experimental data. To that purpose an accelerometer - magnetometer device based on Bosch Sensortec BNO055 was designed. BNO055 is a System in Package (SiP), integrating a 3-axis accelerometer, a 3-axis gyroscope and a 3-axis magnetometer. Accelerometer and magnetometer data were recorded while the device was placed still in several different orientations.

A. Algorithm Convergence

For five different datasets, the convergence of the proposed algorithm is presented in Figures 1 and 2. The proposed algorithm minimizes the penalty function J to a sufficiently small value for both magnetometer and accelerometer in all the recorded datasets.

In order to evaluate the computational efficiency of the proposed algorithm, we compare the convergence of the optimization problem (6) to that of the proposed optimization problem (10). More specifically, both calibration algorithms are applied to the same accelerometer's data. The convergence of each algorithm is evaluated using both penalty functions J_0 and J. Setting a sufficiently small penalty function value as optimization target, the proposed algorithm converges significantly faster as seen in Figure 3.

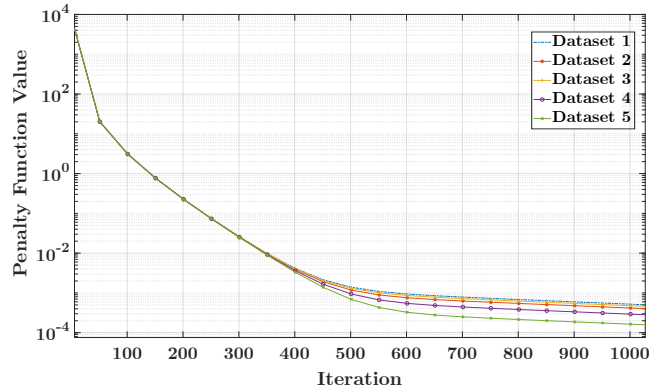

Fig. 1: Algorithm convergence for accelerometer data.

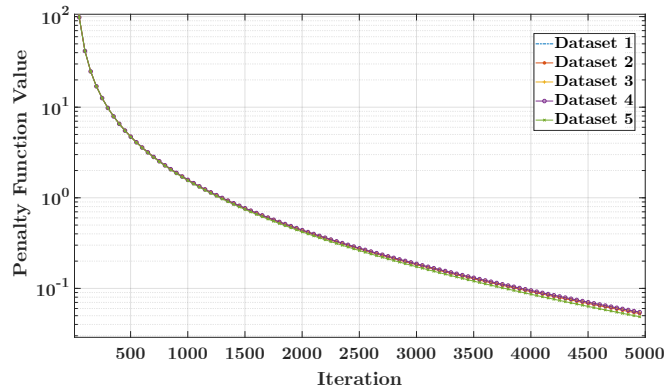

Fig. 2: Algorithm convergence for magnetometer data.

B. Calibration Results

Calibration is based on the fact that the measured gravity and magnetic field should be constant and independent of the sensors' orientation. In figures 4 and 5, the normalized magnitude of both accelerometer's and magnetometer's measurements, before and after calibration, is presented.

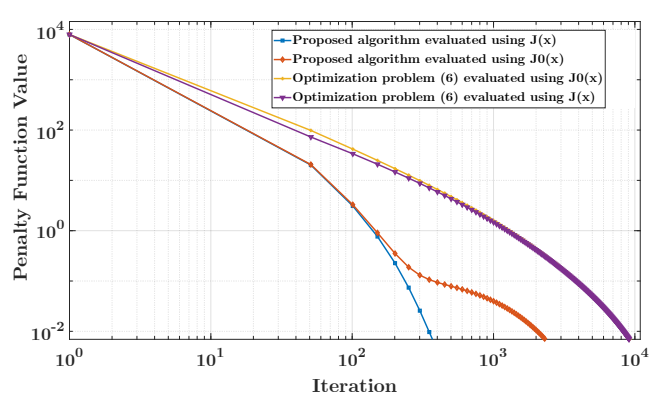

Fig. 3: Comparison between convergence of proposed optimization problem and optimization problem (6)

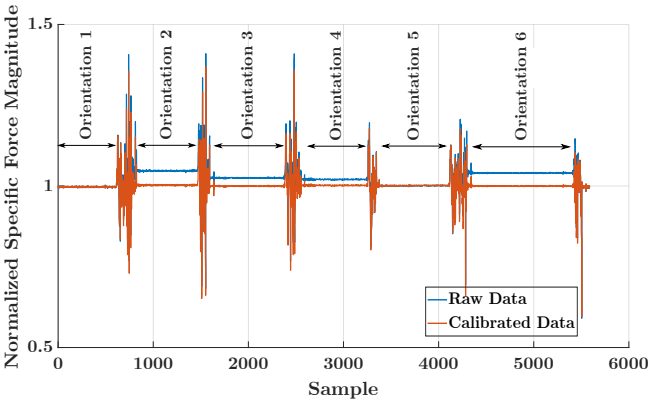

Fig. 4: Normalized magnitude of accelerometer's measurements before and after calibration

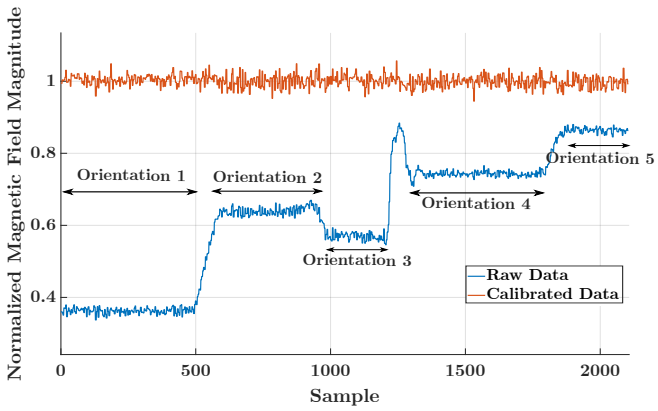

Fig. 5: Normalized magnitude of magnetometer's measurements before and after calibration

V. Conclusion

A new calibration algorithm for 3-axis accelerometers and magnetometers is presented. It achieves significantly faster convergence and higher computational efficiency compared to a popular calibration approach. Applying the proposed algorithm does not require any special piece of equipment like a turn-table in order to calibrate an accelerometer or a magnetometer. Experimental data confirmed the algorithm's efficiency and robustness.

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