A Field Theory Approach in EIT with Green's Functions

Christos Dimas[1], Nikolaos Uzunoglou and Paul P. Sotiriadis

Dept. of Electrical and Computer Engineering ,National Technical University of Athens, Greece

[1] chdim@central.ntua.gr

Abstract: A field theory employing Green's functions is used to develop an imaging algorithm of weak conductivity variations in EIT. The algorithm results in a linear system of equations whose solution gives both the potential and the conductivity distributions.

1 Introduction

EIT imaging of soft deep-body tissues, away from the skin, challenges the numerous existing image reconstruction algorithms. Part of the difficulty is to image significantly small inhomogeneous internal conductivity variations without resorting to memory-expensive inverse problem solution ways [1].

2 Methods

2.1 The Forward Problem

Consider a 2D medium with conductivity $\sigma = \sigma(\boldsymbol{r})$ and two point electrodes sourcing and sinking current I and be placed at $\boldsymbol{r}_+$ and $\boldsymbol{r}_-$ respectively. The EIT's equation is

$$\nabla\sigma\nabla V + \sigma\nabla^2 V = I[\delta(\boldsymbol{r}-\boldsymbol{r}_+) - \delta(\boldsymbol{r}-\boldsymbol{r}_-)] \quad (1)$$

where $V = V(\boldsymbol{r})$ is the potential and $\boldsymbol{r}$ is the observing position. Assuming no other current sources are present and integrating over the domain of interest A', with the use of Green's theorem [2], we get the integral equation

$$\mathrm{V}(\boldsymbol{r}) = \iint_{A'} G(\boldsymbol{r},\boldsymbol{r}')\frac{\nabla\sigma(\boldsymbol{r}')}{\sigma(\boldsymbol{r}')}\nabla V(\boldsymbol{r}')dA' + \frac{I[G(\boldsymbol{r},\boldsymbol{r}_+) - G(\boldsymbol{r},\boldsymbol{r}_-)]}{\sigma_0} \quad (2)$$

Where $\boldsymbol{r}'$ is the position vector within A' where integration takes place. Let σ_0 be the homogeneous conductivity reference and G be a Green's function solution of the homogeneous Laplace's equation $\nabla^2 G(\boldsymbol{r},\boldsymbol{r}') = \delta(\boldsymbol{r}-\boldsymbol{r}')$. Assuming a circular area of radius R_0 and keeping the 1st Fourier series term, the Green's function gives

$$\mathrm{G}(\boldsymbol{r},\boldsymbol{r}') = \frac{1}{2\pi\sigma_0}\left[\ln\frac{1}{|\boldsymbol{r}-\boldsymbol{r}'|} - \frac{rr'}{R_0^2}cos(\varphi-\varphi')\right] \quad (3)$$

Where $r = \|\boldsymbol{r}\|$, $r' = \|\boldsymbol{r}'\|$ and φ, φ' are the angles of $\boldsymbol{r}, \boldsymbol{r}'$ respectively. For small conductivity variation e.g. when $|\sigma - \sigma_0| < \sigma_0/10$ it is $\nabla V \approx \nabla V_0$ and the homogeneous potential V_0 equals the Green function's solution.

2.2 The Inverse Problem

To solve the inverse problem near the centre, the internally inscribed orthogonal parallelogram of the circle is discretized to square pixels, with $\Delta\alpha$ side length. Each pixel corresponds to a conductivity contribution $\sigma_{i_2 j_2}$ such that

$$\ln\sigma_{i_2 j_2} = \sum_{i1=1}^{M}\sum_{j1=1}^{N} a_{i_1 j_1} e^{-\frac{(x_{i1}-x_{i2})^2 + (y_{j1}-y_{j2})^2}{D^2}} \quad (4)$$

Where $\boldsymbol{r}' = (x_{i2}, y_{j2})$ are the central point coordinates of an arbitrary pixel and $\boldsymbol{r_{central}} = (x_{i1}, y_{j1})$ are the central coordinates of the referring pixel. Parameter D must be small enough to avoid aliasing. Taking the gradient of Eq. (4) and using it in Eq. (2) we replace the non-linear term $\nabla\sigma/\sigma$. Using the midpoint integral rule, the inhomogeneous part of equation 2 is written as

$$V(\boldsymbol{r})_{change} = \sum_{j2=1}^{N}\sum_{i2=1}^{M} \varDelta\alpha^2 G(\boldsymbol{r},\boldsymbol{r}')\left.\frac{\nabla\sigma}{\sigma}\right|_{i2,j2} \nabla V_o(\boldsymbol{r}') \quad (5)$$

In Eq. (5), vector $\boldsymbol{r}$ is the position of the voltage observation points. Taking into consideration the homogeneous and inhomogeneous model measurements, the problem concludes to a linear system of equations $M\boldsymbol{x} = b$, where b is the measurement vector, $\boldsymbol{x}$ is the unknown vector of the coefficients $a_{i_1 j_1}$ and the entries of M are

$$m_{kli1j1} = \sum_{i_2=1}^{N}\sum_{j_2=1}^{M} \varDelta\alpha^2 [G(\boldsymbol{r_l},\boldsymbol{r}') - G(\boldsymbol{r_{l+1}},\boldsymbol{r}')]\, \nabla\ln(\boldsymbol{r}')|_{i_2,j_2} \nabla V_o(\boldsymbol{r_{k,k+1}},\boldsymbol{r}') \quad (6)$$

Where $k, k+1$ refer to the current electrode pairs and $l, l+1$ refer to the voltage pairs. In order to have a well-defined problem, the total number of pixels is chosen in such a way to equal the number of the total measurements. Since M is close to singular, the system is solved using the biconjurate gradient's method with preconditioner [3].

3 Results

For the testing, inhomogeneous models were created using the FEMM along with the MATLAB tool. The electrode measurements were computed using Eq. (2) assuming opposite strategy with 32 electrodes and a reconstruction performed using the back-projection algorithm. Then, conductivities were calculated using the method described.

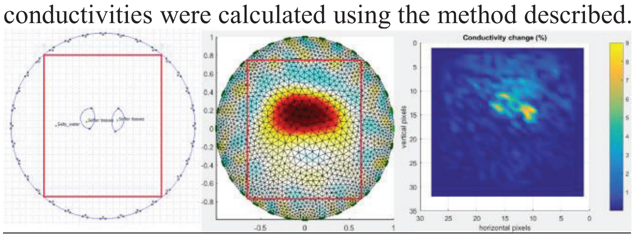

Figure 1: Image reconstruction of two small conductivities near the centre: EIDORS (centre), described approach (right).

4 Conclusion

This approach of EIT conductivity imaging provides a relatively simple and alternative way to image deep-body soft tissue conductivity variations. Applications could be developed in chest imaging and breast cancerous inhomogeneities detection.

References

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