Spurs-Free Single-Bit-Output Frequency Synthesizers For Fully-Digital RF Transmitters

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***Abstract*—Fully-digital single-bit-output (DAC-less) frequency synthesizers with imbedded modulation capability are considered as the foundation for building fully-digital RF transmitters. The paper presents an overview of recent results and introduces a new architecture based on sigma-delta like feedback noise shaping.**

Keywords—Dithering, Direct digital synthesis, frequency spurs, noise shaping, quantization, spectrum

I. INTRODUCTION

Replacing analog RF blocks with digital intensive ones has become a design trend over the past few years [1]-[4]. This is motivated by the increasing challenges in the design of classical analog blocks versus the robustness to noise and PVT variation of their digital counterparts, as well as the design automation tools available for the last ones. Advances in architectures and theory for DSP in RF applications have resulted in power and area efficient digital intensive blocks performing RF signal processing.

Frequency synthesis is fundamental in RF systems and has a number of successful digital intensive architectures to show which can be roughly classified into two main groups: the All-Digital PPLs (ADPLL) [1] and the Direct Digital Synthesizers (DDS) [5]. Although both groups have found an enormous number of practical applications, the successful architectures of both of them still use analog and mixed signal blocks like the time-to-digital converter [1], the VCO and the DAC [5], which need very careful design in the particular IC technology the circuit is fabricated.

Efforts to develop fully-digital frequency synthesizers offering all advantages of digital circuits date at least three decades back [6]-[8]. A way to get such architectures is to remove the DAC in DDS (Fig. 1), i.e., the only analog/mixed-signal block in traditional DDS and use the MSB of the LUT as the output. This leads to the Finite State Machine (FSM) with single-bit (SB) output shown in Fig. 2, often called Pulse-DDS (PDDS) (although the term is better suited to the structure without the LUT [7]). Maintaining the direct form of DDS, i.e. without using an oscillator, this approach has the inherent challenge of *generating a SB digital output with the desirable sinewave-like spectrum, and doing so using only the reference clock* [6],[9].

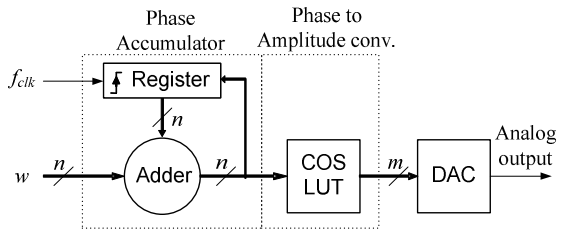

Figure 1: Basic form of a Direct Digital Synthesizer (DDS).

With the exception of generating an output frequency equal to an integer fraction of the clock, the output waveform is irregular with high deterministic jitter and a spectrum full of strong frequency spurs which can be very close to the carrier (as shown in Fig. 4). This general problem of FSM type architectures with SB output demands the use of additional techniques for suppressing the strong spurious signals. Several such ones have been proposed [9] with random dithering being the only purely digital one. The problem of generating spurious-free output with a dithered PDDS has been studied in abstract form in [10]. It is shown that random dithering having appropriate statistics results in spurs-free output [10][11]. Yet, in some sense, the dithering converts the frequency spurs into a noise floor. It is also shown that if one allows some spurs (outside the useful bandwidth) the noise floor can be reduced [10].

This paper reviews PDDS-based frequency synthesis architectures with emphasis on dithering techniques used to eliminate the spurs. Amplitude modulation aspects are discussed as well as approaches for reducing the noise floor associated with the dithering. Finally, a new architecture for fully-digital frequency synthesis is introduced based on sigma-delta like feedback noise shaping.

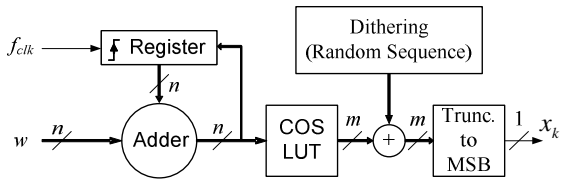

Figure 2: PDDS with amplitude random dithering.

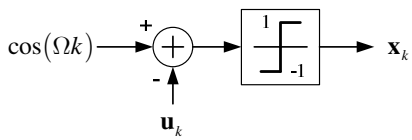

Figure 3: Dithered 1-Bit quantized sinewave capturing PDDS' behavior

II. ELIMINATING SPURS WITH DITHERING

Consider the DDS and the PDDS schemes in Figs. 1 and 2 respectively. The finite resolution of the output of the LUT introduces a representation error of the cosine function in the order of $-6m$ dBc [5]. This error is negligible compared to that introduced by the crude SB (MSB) quantization at the output in Fig. 2. This motivates the abstract model in Fig. 3, which is approximately equivalent to that in Fig. 2 and whose accuracy improves with m.

The phase of the cosine in Fig. 3 is Ωk, where $k \in \mathbb{Z}$ is the discrete time index. With n-Bit phase-accumulator it is $\Omega = 2\pi w / 2^n$ and $w < 2^{n-1}$ is the frequency control word. Note that the accumulator's value is $Ph_k = (kw) \bmod 2^n$ and the LUT implements the function $\cos\left(Ph_k / 2^n\right) = \cos(\Omega k)$. The quantization in Fig. 2 is practically equivalent to applying the signum function, i.e., $\mathbf{x}_k = \operatorname{sgn}\left(\cos(\Omega k) - \mathbf{u}_k\right)$, as in Fig. 3. Also, since the desirable output is $\cos(\Omega k)$ we define the error $\mathbf{e}_k \triangleq \mathbf{x}_k - \cos(\Omega k)$ and so $\mathbf{x}_k = \cos(\Omega k) + \mathbf{e}_k$.

The dithering sequence $\{\mathbf{u}_k\}$ in Fig. 3 is assumed random. It is interesting however to consider first the case without dithering, i.e. when $\mathbf{u}_k = 0$ for all k. Then, with the exception of values of w which are powers of 2, the output $\mathbf{x}_k$ is full of spurs. Specifically the spurs are at frequencies $\omega_\ell = \Omega + 2\pi\ell \cdot \gcd\left(2w, 2^n\right) / 2^n$ with $\ell \in \mathbb{Z}$ [10]. Fig. 4 shows the output for $w = 25$ and $n = 6$.

Now, if $\{\mathbf{u}_k\}$ is a random sequence, the output $\mathbf{x}_k$ is random and *not* wide-sense stationary. To define the spectrum of $\{\mathbf{x}_k\}$ we derive the Time-Average Autocorrelation (TAA) function, $\overline{r}_{\mathbf{x}}(k) = \lim_{M\to\infty} \frac{1}{2M+1} \sum_{m=-M}^{M} E\{\mathbf{x}_{k+m}\mathbf{x}_m\}$, [12] and take the Discrete-Time Fourier Transform (DTFT) of it, $s_{\mathbf{x}}(\omega) = \sum_{k=-\infty}^{\infty} \overline{r}_{\mathbf{x}}(k) e^{-ik\omega}$. The TAA and spectrum definitions are applied similarly to all random and deterministic sequences we encounter [12].

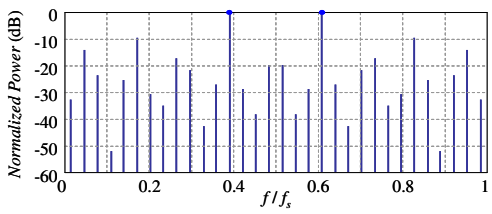

Figure 4: Spectrum of undithered PDDS with $w = 25$ and $n = 6$.

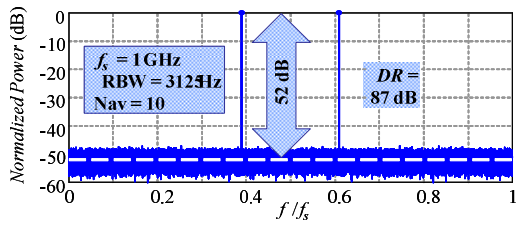

Figure 5: Spectrum with random $\{\mathbf{u}_k\}$ IID uniformly distributed in $[-1,1]$. And $w = 25$, $n = 6$, $f_S = 1\,GHz$, RBW = 3125 Hz, NAV=10.

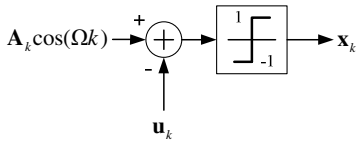

Figure 6: Dithered single-bit quantized amplitude-modulated sinewave

From here on we assume that the dithering sequence $\{\mathbf{u}_k\}$ is formed of independent random variables uniformly distributed in $[-1,1]$. In this case it can be shown [13][14] that $\overline{r}_{\mathbf{x}}(k) = \overline{r}_d(k) + \overline{r}_{\mathbf{e}}(k)$ where $\overline{r}_d(k) = \cos(\Omega k)/2$ is the TAA of the *desirable* output $\cos(\Omega k)$ and $\overline{r}_{\mathbf{e}}(k) = \delta_k / 2$ is that of the error $\{\mathbf{e}_k\}$. Then the output spectrum is

$$s_{\mathbf{x}}(\omega) = s_d(\omega) + s_{\mathbf{e}}(\omega). \tag{1}$$

Writing the discrete-time angular frequency as $\omega = 2\pi f / f_s$ where f_s is the sampling (clock) frequency, it can be shown that $s_d(2\pi f / f_s) = \frac{1}{4}\sum_{k=-\infty}^{\infty} \delta\left(f / f_s \pm \Omega/(2\pi) - k\right)$ and $s_{\mathbf{e}}(\omega) = 1/2$, so the error spectrum is flat[1], Fig. 5, [10].

The output is spurs-free and the Dynamical Range (DR), defined as the ratio of the power of the desirable signal to the PSD of the near-in noise $s_{\mathbf{e}}$ is $DR = 10\log_{10}(f_S) - 3.01$ dB [13]. Note that the dynamic range observed in Fig. 5, and in a spectrum analyzer, is $10\log_{10}(RBW)$ dB below DR due to the Resolution Bandwidth (RBW) of the measurement, i.e., in Fig. 5 it is $87 - 10\log_{10}(3125) = 52$.

Amplitude modulation (AM) is achieved by inserting a digital multiplier immediately after the LUT in Fig. 2. The situation is modeled as in Fig. 6 where $\cos(\Omega k)$ is replaced by $A_k \cos(\Omega k)$. If $\{A_k\}$ is deterministic and as long as $|A_k| \le 1$ and $\{A_k\}$ is band-limited by Ω_{A} with $\Omega_{\mathrm{A}} < \Omega < \pi - \Omega_{\mathrm{A}}$, then again $s_{\mathbf{x}}(\omega) = s_d(\omega) + s_{\mathbf{e}}(\omega)$ where s_d is the spectrum of $A_k \cos(\Omega k)$ defined as before [14].

[1] We assumed that PDDS' continuous-time output is a sequence of Dirac functions. If square or other pulses are used, the spectrum must be weighted using their corresponding Fourier transforms.

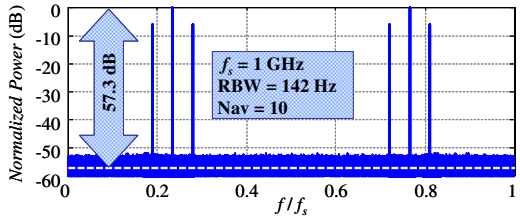

Figure 7: The spectrum when $A_k = 1/2 + (1/2)\sin(\Omega_{\text{mod}} k)$

Moreover $s_{\mathbf{e}}(\omega) = 1 - \bar{A}^2/2$, where $\bar{A}$ is the RMS value of $\{A_k\}$. So, the noise floor is flat again and the DR is defined accordingly [14]. Fig. 7 shows the spectrum when $w = 15$, $n = 6$, $A_k = 1/2 + (1/2)\sin(\Omega_{\text{mod}} k)$, $\Omega_{\text{mod}} = 2\pi \cdot 0.045$ and $f_s = 1GHz$, RBW = 142Hz and averaging NAV=10.

When $\{\mathbf{A}_k\}$ is a wide-sense stationary random sequence, with $r_{\mathbf{A}}(n) \triangleq E\{\mathbf{A}_{n+m}\mathbf{A}_m\}$, similarly we get that $s_{\mathbf{x}}(\omega) = s_d(\omega) + s_{\mathbf{e}}(\omega)$, where s_d is the spectrum of the random sequence $\{\mathbf{A}_k \cos(\Omega k)\}$. Again the output is spurs-free with $s_{\mathbf{e}}(\omega) = 1 - r_{\mathbf{A}}(0)/2$, [14].

III. Enhancing the Dynamic Range

One way to reduce the level of the noise floor introduced by the dithering is to use two opposite-dithering quantization paths, following the cos LUT, and add the outputs as shown in Fig. 8. The random sequence $\{\mathbf{u}_k\}$ is again IID and uniformly distributed in $[-1,1]$. The output signal is $\mathbf{x}_k = \text{sgn}(\cos(\Omega k) - \mathbf{u}_k) + \text{sgn}(\cos(\Omega k) + \mathbf{u}_k)$ and it can be shown that when $w \neq 2^{n-2}$ (which is a trivial case) the dynamic range is $DR \cong 10\log_{10}(f_s) + 2.61$ (dB), i.e. about 5.7

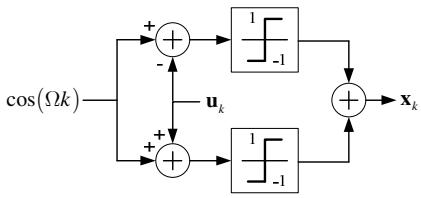

Figure 8: Averaging the outputs of two PDDS with opposite dithers

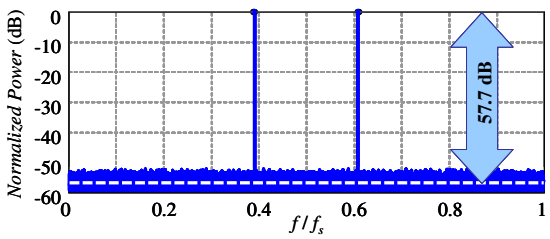

Figure 9: Spectrum with opposite dithering. $w = 25$, $n = 6$, $f_S = 1GHz$, RBW = 3125 Hz, NAV=10.

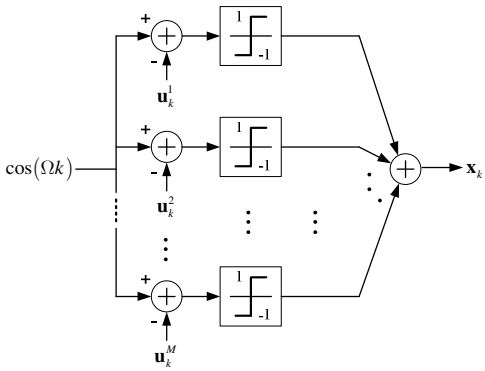

Figure 10: Multi-path PDDS with independent dithers

dB higher than in the case of Fig. 3, [13]. The corresponding spectrum is shown in Fig. 9 where $DR = 57.7$ dB.

Another similar scheme reducing the noise floor is shown in Fig. 10. Here M dithering and quantization paths following the LUT, with mutually independent, IID and uniformly distributed in $[-1,1]$ random sequences $\{\mathbf{u}_k^j\}$, $j = 1,2,..,\text{M}$ are used. The scheme results in dynamic range $DR = 10\log_{10}(f_s) + 10\log_{10}(M) - 3.01$ (dB), an improvement of $10\log_{10}(M)$ dB compared to that of Fig. 3, [13].

Finally, colored uniformly distributed dither instead of white (IID) one can result in spurs-free output with slightly higher DR, e.g. about 2.5 dB in Fig. 11, [13].

IV. Sigma-Delta Single-Bit Frequency Synthesis

The problem with random dithering for spurs elimination/suppression in SB quantization of the sinewave is that the power of the dither spreads in the whole spectrum via the strong nonlinearity of the signum function, Fig. 3. The last one acts as a strongly nonlinear mixer of the dither with the sinewave, and, frequency shaping of the dither can only slightly improve the noise floor near in as shown in Fig. 11.

A new sigma-delta type architecture for SB frequency synthesis is illustrated in Fig. 12, allowing for reduced noise (and spurs) near in. The concept here is that the ideal spectrum of the output $\mathbf{x}_k = \text{sgn}(\cos(\theta_k))$ is that of $\cos(\Omega k)$ and so orthogonal to $\sin(\Omega k)$. Any other components of $\mathbf{x}_k$ must be removed, at least near-in. To this end $\mathbf{x}_k$ is

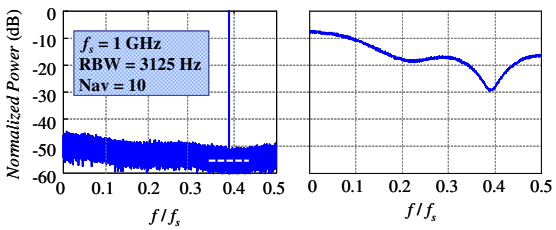

Figure 11: Colored-dithered PDDS output spectrum (left) . Spectrum of the colored, uniformly distributed dither (right).

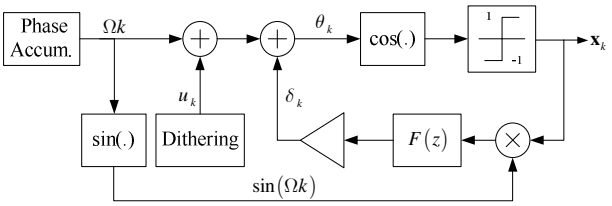

Figure 12: A sigma-delta single-bit frequency synthesizer

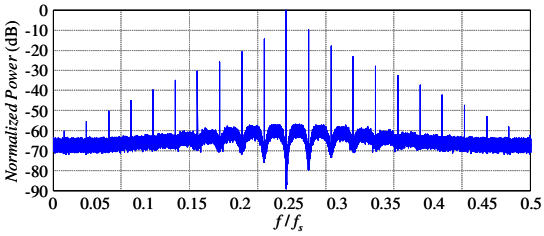

Figure 13: Spectrum of the sigma-delta single-bit frequency synthesizer $w = 8001$ and $n = 15$, $f_S = 1\,GHz$, RBW = 3391 Hz, NAV = 14.

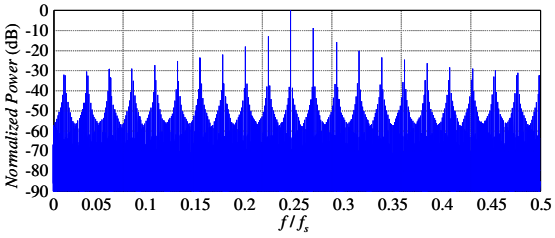

Figure. 14: Spectrum of undithered PDDS with $w = 8001$ and $n = 15$.

multiplied by $\sin(\Omega k)$, integrated and low-pass filtered by $F(z)$ generating feedback signal δ_k which is added to the phase of the cosine.

For $\Omega = 2\pi w / 2^n$ with $w = 8001$ and $n = 15$ the output spectrum is shown in Fig. 13 whereas the spectrum or $\mathrm{sgn}(\cos(\Omega k))$ is shown in Fig. 14 for comparison.

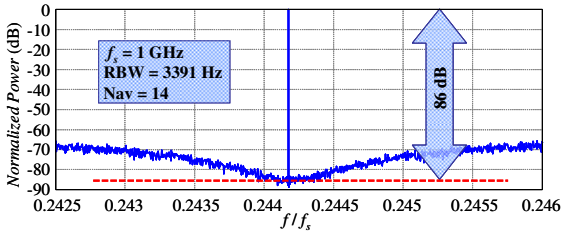

Figure 15: Zoom in of the spectrum of the Σ/Δ SB frequency synthesizer

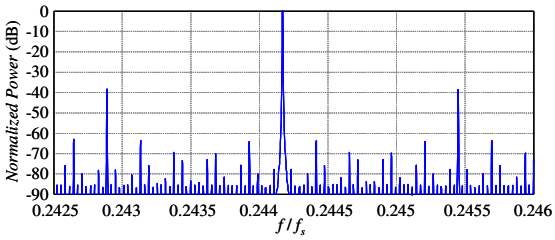

Figure. 16: Zoom in spectrum of undithered PDDS, $w = 8001$, $n = 15$.

Figs. 15, 16 are zoom-in versions of Figs. 13, 14 respectively demonstrating the near-in spectrum. Note that the spectrum in Fig. 15 is taken with RBW = 3391 and so the near-in DR is in principle 35.3 dB higher than measured, i.e. about 121dB. Yet, this must be verified with more simulation as spurs may be hidden under the noise floor in Fig. 15.

V. Conclusions

Fully-digital single-bit-output (DAC-less) frequency synthesis, based on Pulse-Direct Digital Synthesizer can achieve spurs-free spectrum if the appropriate random dithering is used. In the simplest case, a single dither and quantization path is used resulting in dynamic range of $10\log_{10}(f_S) - 3.01$ dB. The dynamic range is improved by 5.7 dB if two quantization paths with opposite dithers and averaging at the output are used. Multiple (M) quantization paths with independent dithers and averaging at the output result in $10\log_{10}(M)$ dB improvement. To improve the dynamic range near-in, a new architecture based on feedback sigma-delta type noise shaping was introduced achieving more than 30 dB of improvement near-in.

Acknowledgement

Described work was partially supported by Broadcom Foundation USA

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