Optimizing Continuous-Time Filters Driven by Bang-Bang Signals

Paul P. Sotiriadis

***Abstract*—This work introduces an optimization method to minimize the noise and the power consumption of active filters driven by bang-bang signals taking into account the hard voltage-range constraints of the amplifiers and guaranteeing their full-range operation without saturation.**

I. Introduction

Although active continuous-time filters have been studied extensively for decades, almost all of the efforts have followed frequency-domain approaches or have been based on sinusoidal test-signals otherwise, e.g., [1]-[10]. This leaves some unanswered questions in the design and optimization of active filters in the presence of hard time-domain constraints.

This work lays out the basics of a new approach to optimize active continuous-time filters driven by bang-bang signals, i.e. of the form $u(t) \in \{\pm V_u\}$ for all $t \in \Re$.

There are many cases where active continuous-time filters are driven exclusively by bang-bang signals, or, the input is allowed to be a bang-bang signal of maximal amplitude. Such cases include: the filtering of the output of digital-to-analog converters, the analog filtering of pulse-width modulated signals, the operation of sigma-delta modulators, any (asynchronous) loop with a comparator preceding an active continuous-time filter and the analog filtering of continuous-time digital signals [11]-[12].

This work is motivated by the inadequacy of the classical filter optimization techniques, based on sinewave excitation, or, using the Euclidean norm $\|.\|_2$ in the time or frequency domains, to treat signals and filters with hard constraints of the form $-V_M \leq v_i(t) \leq V_M$, for some voltage V_M and all $t \in \Re$. Such constraints are natural in active filters due to the input and output voltage swing range of transconductance and operational amplifiers. One can also choose voltage V_M below the hard limit to bound the distortion (e.g. harmonic) of the amplifier [4], [13].

The above constraints raise two important questions: 1) What is the maximum absolute input voltage, V_u, such that all node voltages of the filter satisfy $-V_M < v_i < V_M$, all the time? 2) Among the infinitely many combinations of values of the filter's elements resulting in the same transfer function, which one results in the filter with the maximum V_u, the minimum output noise and minimum power consumption?

This paper treats the above problems in the case of active filters driven by bang-bang signals. It illustrates first the inadequacy of the sinewave excitation approach to provide the answers for the above questions; then it introduces a new metric that provides the exact maximum (and minimum) of the filter's nodes voltages; it uses this metric to introduce a new optimization method for minimum noise and power consumption based on nodes (state) voltages balancing.

To simplify the presentation, the theory is illustrated in the class of low-pass Gm-C filters whose output is a node voltage. The theory can be extended to other classes of active filters with minimal effort. Simulation results demonstrate the performance improvements.

II. Bang-Bang Vs. Sinusoidal Input

This section introduces a metric for determining the voltage range at the nodes of active filters driven by bang-bang signals, leading to a new power and noise optimization methodology. The metric is introduced through an example of a Gm-C filter and is compared to the classical sinewave-input based characterization which is shown to be inadequate for filters driven by bang-bang signals.

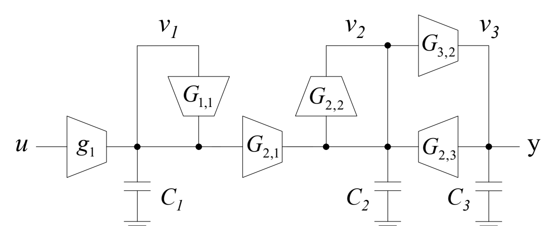

Fig. 1. A third order Low-Pass Chebyshev Gm-C. Filter

Consider the third order, Chebyshev, low-pass Gm-C filter[1] in Figure 1. It has cut-off frequency $f_c = 1$MHz and 3dB in-band ripple. The capacitors are chosen to be $C_1 = C_2 = C_3 = 1$pF and the input transconductance is $g_1 = 1.72\mu$A/V. Defining the matrices and vectors

$$\mathbf{G} = [G_{i,j}]_{i,j=1}^{3} = \begin{bmatrix} -1.88 & 0 & 0 \\ 6.28 & -1.88 & -5.76 \\ 0 & 5.76 & 0 \end{bmatrix} \frac{\mu A}{V} \quad (1)$$

$$\mathbf{C} = \begin{bmatrix} C_1 & 0 & 0 \\ 0 & C_2 & 0 \\ 0 & 0 & C_3 \end{bmatrix}, \ \mathbf{g} = \begin{bmatrix} g_1 \\ 0 \\ 0 \end{bmatrix} \text{ and } \mathbf{v} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} \quad (2)$$

[1]It was derived using MATLAB's *Cheby*1 function.

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we can write the filter's state-space equations as

$$\mathbf{C}\dot{\mathbf{v}} = \mathbf{G}\mathbf{v} + \mathbf{g}u \tag{3}$$

$$y = (0,0,1)\mathbf{v} \tag{4}$$

The magnitudes of the transfer functions $H_j(s) \triangleq V_j(s)/U(s)$ from input voltage u to the node (state) voltages v_j, $j = 1,2,3$ are shown in Figure 2. Here the output coincides with v_3 and so the filter's transfer function is $H(s) = H_3(s)$.

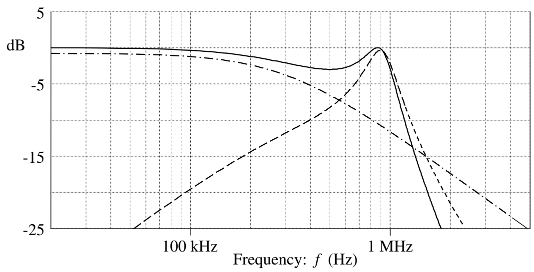

Fig. 2. Magnitudes of the Transfer Functions: H_1 in dash-dots, H_2 in dashed line, H_3 in solid line.

A critical piece of information in the design of active filters is the maximum (and minimum) voltages of the nodes, internal ones or not. This is because the input and output voltage ranges of the amplifiers and the power supply voltage impose hard voltage constraints which must be respected otherwise the signal is distorted.

The typical method to estimate the maximum voltage at the i^{th} filter node is to assume a sinusoidal input $u(t) = \sin(2\pi f t)$, calculate the absolute maximum of the input-to-i^{th}-node transfer function, $H_i(f)$, and multiply it by the maximum sinusoidal amplitude allowed at the input. Doing so for the filter in Figure 1 and assuming input amplitude of one we derive

$$\max_{t\,,\,u\,:\,\text{sinusoidal}} |v_i(t)| = \max_f |H_i(f)| \tag{5}$$

for $i = 1,2,3$. The maxima of the transfer functions can be derived from Figure 2; they are:

$$\max_{t\,,\,u\,:\,\text{sinusoidal}} |v_1(t)| = 0.916 \tag{6}$$

$$\max_{t\,,\,u\,:\,\text{sinusoidal}} |v_2(t)| = 0.961 \tag{7}$$

$$\max_{t\,,\,u\,:\,\text{sinusoidal}} |v_3(t)| = 1.000 \tag{8}$$

Therefore, from (6)-(8) we know that an input sinusoidal of amplitude one results in an output sinusoidal of amplitude at most one in all three nodes[2].

Extending this argument to the case of bang-bang signals, of amplitude one, leads to *wrong* conclusions. This is illustrated in Figures 3-5 showing voltages v_1, v_2 and v_3, *each one for a particular bang-bang input signal* $u = u_i(t)$, $i = 1, 2$ and 3 respectively.

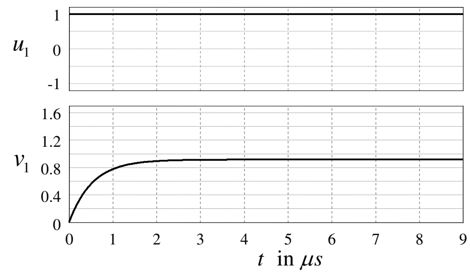

Fig. 3. Top: Bang-Bang input $u = u_1$ maximizing v_1. Bottom: v_1.

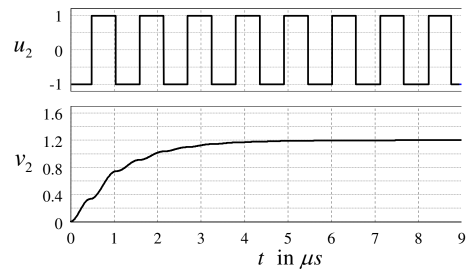

Fig. 4. Top: Bang-Bang input $u = u_2$ maximizing v_2. Bottom: v_2.

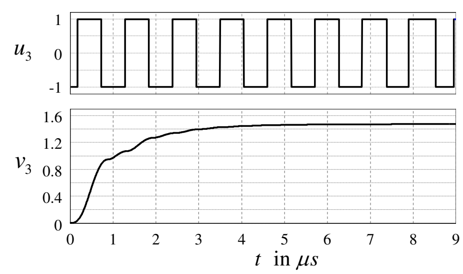

Fig. 5. Top: Bang-Bang input $u = u_3$ maximizing v_3. Bottom: v_3.

It can be shown that the maximum[3] value the state voltage v_i can achieve for ± 1 bang-bang inputs is

$$\sup_{t,u:\text{Bang-Bang}} |v_i(t)| = \int_0^\infty |\mathbf{e}_i^T e^{\mathbf{C}^{-1}\mathbf{G}\,t}\mathbf{C}^{-1}\mathbf{g}|\,dt, \tag{9}$$

which, in the case of the example above gives:

$$\sup_{t\,,\,u\,:\,\text{Bang-Bang}} |v_1(t)| = 0.917 \tag{10}$$

$$\sup_{t\,,\,u\,:\,\text{Bang-Bang}} |v_2(t)| = 1.213 \tag{11}$$

$$\sup_{t\,,\,u\,:\,\text{Bang-Bang}} |v_3(t)| = 1.472 \tag{12}$$

Comparing (6)-(8) to (10)-(12) we see that the nodes' voltages can reach significantly higher values in the case of bang-bang inputs than those in the case of sinusoidal inputs. In this third-order filter, bang-bang inputs can result in 47% higher voltages. The percentage increases to

[2]The maximum is achieved for at least one frequency f by v_3. The above statements are valid only in steady state operation.

[3]To be more accurate, *maximum* should be considered as *suppremum*.

70%, 91%, 109%, 127% and 144% for similar Chebyshev filters of orders $n = 4, 5, 6, 7$ and 8 respectively (derived using MATLAB's command "cheby1"). Therefore, the classical approach for deriving the maxima of the absolute nodes' voltages cannot be used.

A. Maximum Nodes Voltages and Power Consumption

Using (9) we define the suprema of the absolute nodes voltages, i.e.,

$$V_{M\,i} = \sup_{t\,,\,u\,:\,\text{Bang-Bang}} |v_i(t)| \tag{13}$$

for $i = 1, 2, \ldots, n$ and set

$$V_M = \max_{i=1,2,\ldots,n} V_{M\,i} \tag{14}$$

to be the maximum of the suprema. E.g. in the example above it is $V_M = 1.472$V.

Since a bang-bang ± 1 input signal (this is the class of input signals we consider here) can bring at least one of the nodes' voltages arbitrarily close to V_M, the input and output voltage ranges of the transconductors should be at least equal to V_M. Similarly the power supply voltage should be at the minimum equal to $\pm(V_M + V_R)$ where V_R is the voltage room required by the transconductors' topology in the utilized technology.

Note that although the amplifiers in the filter have certain hard input and output voltage limits, they introduce distortion even for input / output voltages below these limits [1], [13]. Therefore, V_R can be deliberately chosen larger than the minimum possible in order to keep signal distortion below a certain level.

The current drawn from the power supply by the transconductors is approximately proportional to their gain [2], [4]. Following the discussion above, the power consumption of the filter is approximated by (15) where η is an appropriate constant.

$$P \;=\; \eta\,(V_M + V_R)\left(\sum_{i,j=1}^{n} |G_{i,j}| + \sum_{i=1}^{n} |g_i|\right) \tag{15}$$

III. Filter Optimization Via State Balancing

So far we have captured the power consumption of the filter via (15) which can also be used, to a certain extent, to bound the distortion of the amplifiers through the choice of V_R.

Filter's chip-area is approximated by the total area occupied by the capacitors. To simplify the presentation of the proposed optimization method we choose all capacitors to have the same and fixed size and so the total chip area remains constant. An additional simplifying assumption, valid for many classes of low-pass Gm-C filters, is that the last state of the filter is also the output voltage, i.e. $y = v_n$.

The noise power at filter's output, introduced by a transconductor, g, whose output is connected to the i^{th} node of the filter is given by

$$\overline{V^2_{n_{i,o}}} = 4kT\gamma|g| \int |H_{i,o}(f)|^2 df. \tag{16}$$

where $H_{i,o}$ is the tranfer function from the total current sinking into (or drained from) the i^{th} node, to the output node voltage y. T is the absolute temperature, k is Boltzmann's constant and γ is the excess noise factor of the transconductor topology. The total noise contribution from all transconductors is

$$\overline{V^2_{n_o}} = 4kT\gamma \sum_{i=1}^{n}\left[\sum_{j=1}^{n}|G_{i,j}| + |g_i|\right]\int |H_{i,o}(f)|^2 df \tag{17}$$

This work proposses the voltage state balancing, equation (18), as an optimization method for filters driven by bang-bang input signals,

$$\mathbf{v} = \mathbf{T}^{\alpha}\,\bar{\mathbf{v}} \tag{18}$$

where $\alpha \in \Re$ and $\mathbf{T}$ is the positive diagonal matrix

$$\mathbf{T} = \frac{1}{V_{M\,n}}\begin{bmatrix} V_{M\,1} & 0 & \ldots & 0 \\ 0 & V_{M\,2} & \ldots & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & \ldots & V_{M\,n} \end{bmatrix}. \tag{19}$$

It is shown in the following examples that $\alpha = 1$ is the desirable value, i.e., the transformation is essentially $\mathbf{v} = \mathbf{T}\,\bar{\mathbf{v}}$, however, it is worth examining the performance of the transformed filter with α ranging within $[-\epsilon, 1+\epsilon]$, for a small $\epsilon > 0$.

Transformation (18) implies the change of the transconductance matrix, $\mathbf{G} \mapsto \mathbf{T}^{-\alpha}\mathbf{G}\mathbf{T}^{\alpha}$ and the change of the input transconductance vector $\mathbf{g} \mapsto \mathbf{T}^{-\alpha}\mathbf{g}$. Since $\mathbf{T}$ is diagonal the number of transconductors and their connectivity remain unchanged.

IV. Examples - Simulation Results

Since the transfer function (and gain) and the chip area of the filter remain unchanged[4], we need to compare the prototype filter $(\mathbf{C}, \mathbf{G}, \mathbf{g})$ with the transformed one $(\mathbf{C}, \mathbf{T}^{-\alpha}\mathbf{G}\mathbf{T}^{\alpha}, \mathbf{T}^{-\alpha}\mathbf{g})$ with respect to power consumption and output noise power. Moreover, it can be shown that given the considered optimization constraints an appropriate *Figure-Of-Merit* of the transformed filter's performance vs. that of the prototype one, is $1/(P \cdot \overline{V^2_{n_o}})$ which is expressed in dB as

$$\text{FOM} = -10\log_{10}\left(P \cdot \overline{V^2_{n_o}}\right) \tag{20}$$

The presented theory is applied first to a low-pass, fourth order Chebyshev filter with cut-off frequency

[4]...and filter's distortion is accounted via the choice of V_R in (15)

f_c =1MHz and 3dB pass-band ripple. The prototype filter is a direct Gm-C implementation of the state-space matrices derived using MATLAB's command "cheby1".

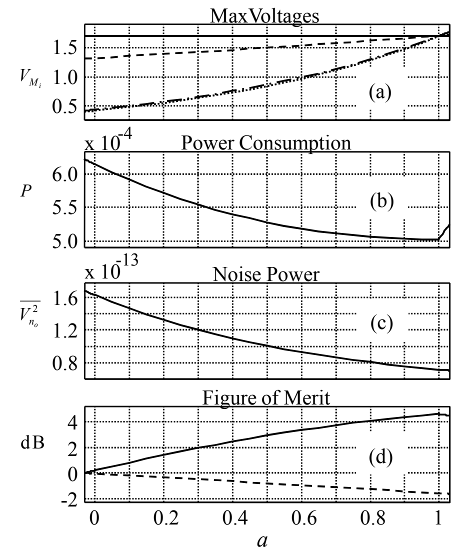

Fig. 6. 4th order Low-Pass Chebyshev Gm-C filter optimization

The results are shown in Figure 6a-d with respect to parameter $\alpha \in [-\epsilon, 1+\epsilon]$. Graph (a) shows the maximum (supremum) voltages of the four nodes, (b) shows the power consumption, (c) shows the output noise power and (d) shows the figure of merit in solid line, given by (20), achieving a maximum of 4.6dB at $\alpha = 1$; the dashed line gives the FOM when frequency-based maximum nodes voltages (equation (5)) are used for state balancing.

Similar results are presented for an eighth order Chebyshev filter with the same parameters as in the one before. Again, the prototype was a direct Gm-C implementation of the state-space matrices derived using MATLAB. In Figure 7d the FOM in solid line achieves a maximum of 12.4dB at $\alpha = 1$; again, the dashed line gives the FOM when frequency-based maximum nodes voltage estimation is used for balancing.

Finally, the proposed methodology can by used to improve the performance of prototype filters that are already optimized using classical methods. E.g. an eighth order Chebyshev filter, [14], with the same parameters as before has room for 2.9dB FOM improvement.

V. Conclusions

A new optimization method was introduced to minimize the noise and power consumption of active filters driven by bang-bang signals. It is based on an exact metric of the maximum achievable voltages of the filter nodes and takes into account the hard input and output voltage-range constraints of the amplifiers guaranteeing their full-range operation without saturation. For presentation simplicity the theory was illustrated in the class of low-pass

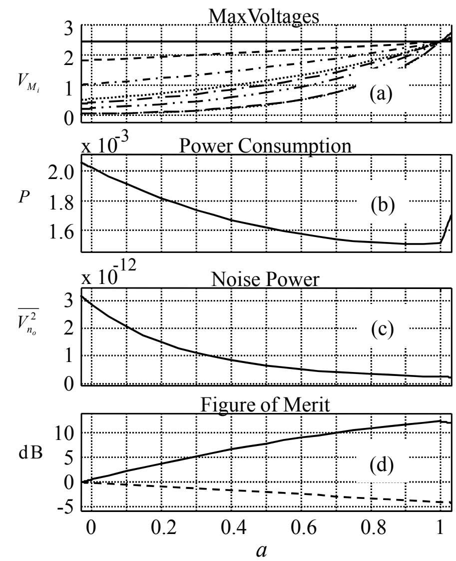

Fig. 7. 8th order Low-Pass Chebyshev Gm-C filter optimization

Gm-C filters whose output is a node voltage. Simulation results demonstrate the performance improvements.

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