

# Intrinsic Jitter of Flying-Adder Frequency Synthesizers

Paul P. Sotiriadis  
 Sotekco Electronics LLC  
 Baltimore, MD, USA  
 pps at iee dot org

**Abstract—Motivated by the “open problem” questions stated recently in [1], this work provides mathematical estimates of the Flying-Adder frequency synthesizer’s deterministic jitter. The analytical results have been verified using MATLAB.**

## INTRODUCTION

Frequency synthesis is important for most digital and high-frequency analog circuits and systems. Digital circuits are driven by the clock that is required to have minimal jitter and frequency and duty cycle within desirable range. Analog circuits, especially communication ones, require a reference-frequency signal with minimal spurs and phase noise.

Chip space and power constraints typically force the use of the simplest frequency synthesizer possible that can meet the requirements.

A new, low-complexity architecture, the Flying-Adder frequency synthesizer [2] has been introduced in the literature recently. It is an elegant loop-less methodology using pulse-swallowing [3-5] to achieve improved frequency resolution.

This paper presents a mathematical approach to the estimation of the intrinsic (deterministic) jitter of the Flying-Adder synthesizer, considered as irregularity of its output pulse sequence. Derivations of the period and average frequency of the synthesizer’s output signal are also derived to allow for the fractional jitter derivation and provide some information about the spectrum of the synthesizer.

## OPERATION OF THE FLYING ADDER SYNTHESIZER

We briefly review the operation of the Flying-Adder synthesizer. Starting from Figure 1b, a family of  $M = 2^m$  periodic, 50% duty-cycle square waves of the same frequency and relative phase-offsets, that form an arithmetic progression with phase step  $-2\pi / M$  (which corresponds to time offset  $\Delta$ ), are generated, typically using a ring oscillator with the corresponding number of stages as shown in Figure 1a.

The  $M = 2^m$  phases are fed into the  $M$ -to-1 multiplexer (MUX) of the Flying-Adder in Figure 2 and the rising edges of MUX’s output, signal  $s(t)$ , trigger the  $n$ -Bit Register changing its value from  $x_k$  to

$$x_{k+1} = (x_k + w) \bmod 2^n.$$

where  $w$  is the  $n$ -bit long frequency control word. To simplify the analysis we can assume that  $x_0 = 0$ . Note that the nonnegative variable  $k$  counts the rising edges of  $s(t)$  and it is the discrete-time reference for the Flying-Adder.

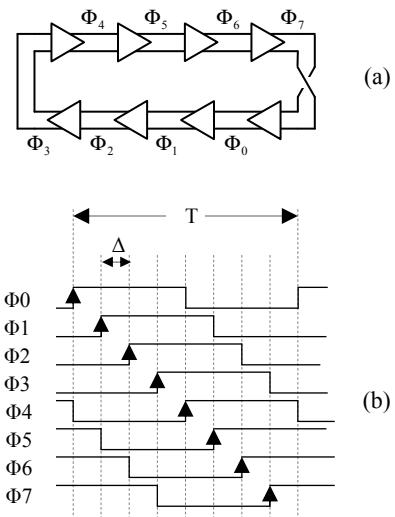


Figure 1: Input Phases to the Flying-Adder

Register’s value,  $x_k$ , is truncated by keeping the first  $m$  most significant bits. This defines the phase-selection variable

$$y_k = x_k \bmod 2^{n-m}$$

which drives the MUX. Signal  $s(t)$ , a sequence of spikes and pulses, is fed to the D-Flip-Flop which acts as a frequency divider by-2 generating an output clock-cycle in  $v(t)$  for every two consecutive rising edges (or spikes) of  $s(t)$ .

### A. Example of Flying-Adder’s Operation

Consider the case where  $n = 4$ ,  $m = 2$  and frequency word is  $w = 7$ . Figure 3 presents the signals in the Flying-Adder. The x-axis is the (real) time axis in multiples of  $\Delta$ . The four input phases,  $\Phi_i$ ,  $i = 1, 2, 3, 4$ , are shown in Figure 3a.

The input phase,  $\Phi_i$ , propagating to the output is selected by the MUX and it is  $s(t) = \Phi_{y_k}(t)$ . The time intervals within which each of the phases  $\Phi_i$ ,  $i = 1, 2, 3, 4$  is selected are indicated with *thick* line segments in Figure 3a.

By definition, the discrete-time  $k$ , shown in Figure 3f, is the result of counting the rising edges in  $s(t)$ . All activity in the Flying-Adder takes place at the rising edges of  $s(t)$ . The discrete-time has value  $k$  between the  $k^{\text{th}}$  and  $k+1$  rising edges.

The sequence  $d_k$ , shown in part Figure 3d, is defined as

$$d_k = (y_k - y_{k-1}) \bmod 2^m$$

and has the property that  $d_k > 0$  if and only if the  $k^{\text{th}}$  rising edge of signal  $s(t)$  results in a change of value from  $y_{k-1}$  to  $y_k$ . Such a change results in a change of the selected phase, and, a  $k+1$  rising edge appearing  $d_k \cdot \Delta$  seconds later. E.g. the 2<sup>nd</sup> rising edge of  $s(t)$  appears at  $t = \Delta$  in Figure 3e. Also, since  $y_1 = 1$  and  $y_2 = 3$ , it is  $d_2 = 2$  and the next rising edge appears at  $t = 3\Delta$ .

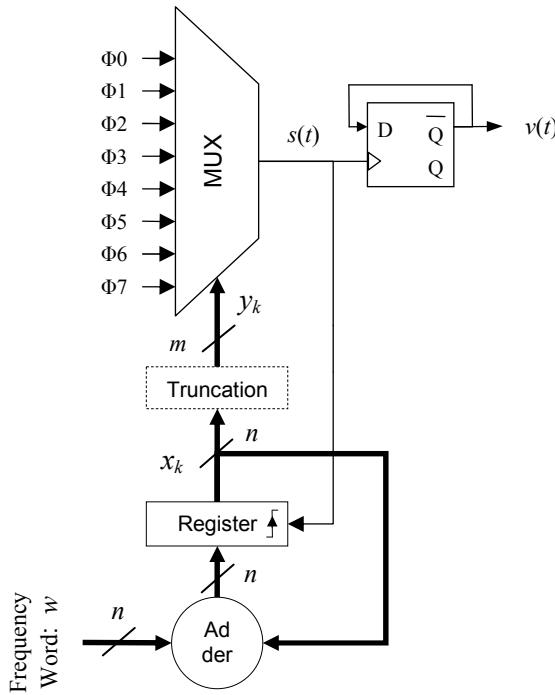


Figure 2: The Flying-Adder architecture (following [5] - with 8 input phases)

Note that there may be consecutive discrete-time values  $k$  for which the value of the phase-control sequence  $y_k$  does not change (e.g. when  $w < 2^{n-m}$ ), i.e.  $y_{k-1} = y_k$  and so it is  $d_k = 0$ . In this case the  $k^{\text{th}}$  rising edge is due to the input phase  $y_k$  and since  $y_{k-1} = y_k$  the  $k+1$  rising edge will be the next rising edge of phase  $y_k$  that appears after  $2^m \Delta$  seconds. Therefore we define

$$\delta_k = \begin{cases} d_k & \text{if } d_k > 0 \\ 2^m & \text{otherwise} \end{cases}$$

and so, in all cases, the time-distance between the  $k^{\text{th}}$  and the  $k+1$  rising edges is equal to  $\delta_k \cdot \Delta$ .

Finally, the D-Flip-Flop counts the spikes (or pulses) in  $s(t)$  modulo-2, (frequency-division by 2) resulting in the output signal  $v(t)$  shown in Figure 3g.

#### DISCRETE AND CONTINUOUS-TIME PERIODS

The following results regarding the periods of the Flying-Adder's signals, stated as theorems without proofs, are important in understanding the structure of the output waveform  $v(t)$ . Note that here, by period of a continuous or discrete time signal we always mean the minimum (fundamental) period of it.

*Theorem 1: The discrete-time period of sequence  $\delta_k$  is*

$$L = \frac{2^{n-m}}{\gcd(w, 2^{n-m})}.$$

Theorem 1 implies that the continuous-time period of sequence  $\delta_k$ , and therefore that of  $s(t)$ , is

$$T_P = \Delta \sum_{k=1}^L \delta_k.$$

*Theorem 2: The continuous-time period of sequence  $\delta_k$ , as well as that of  $s(t)$ , is*

$$T_P = \Delta \cdot \begin{cases} \frac{2^n - (2^m - 1)w}{\gcd(w, 2^{n-m})} & \text{if } 0 < w < 2^{n-m} \\ \frac{w}{\gcd(w, 2^{n-m})} & \text{if } 2^{n-m} \leq w < 2^n \end{cases}.$$

*Theorem 3: The continuous-time period of output  $v(t)$  is*

$$T_v = \Delta \cdot \begin{cases} \frac{2^n - (2^m - 1)w \cdot L}{2^{n-m}} & \text{if } 0 < w < 2^{n-m} \\ \frac{w \cdot L}{2^{n-1}} & \text{if } 2^{n-m} \leq w < 2^n \text{ and } L = 1 \\ \frac{w \cdot L}{2^n} & \text{if } 2^{n-m} \leq w < 2^n \text{ and } L > 1 \end{cases}.$$

The average frequency, of the output  $v(t)$ , defined as the number of cycles in  $v(t)$  within a period  $T_v$  divided by  $T_v$  (see [6]), is derived using the above results. It is

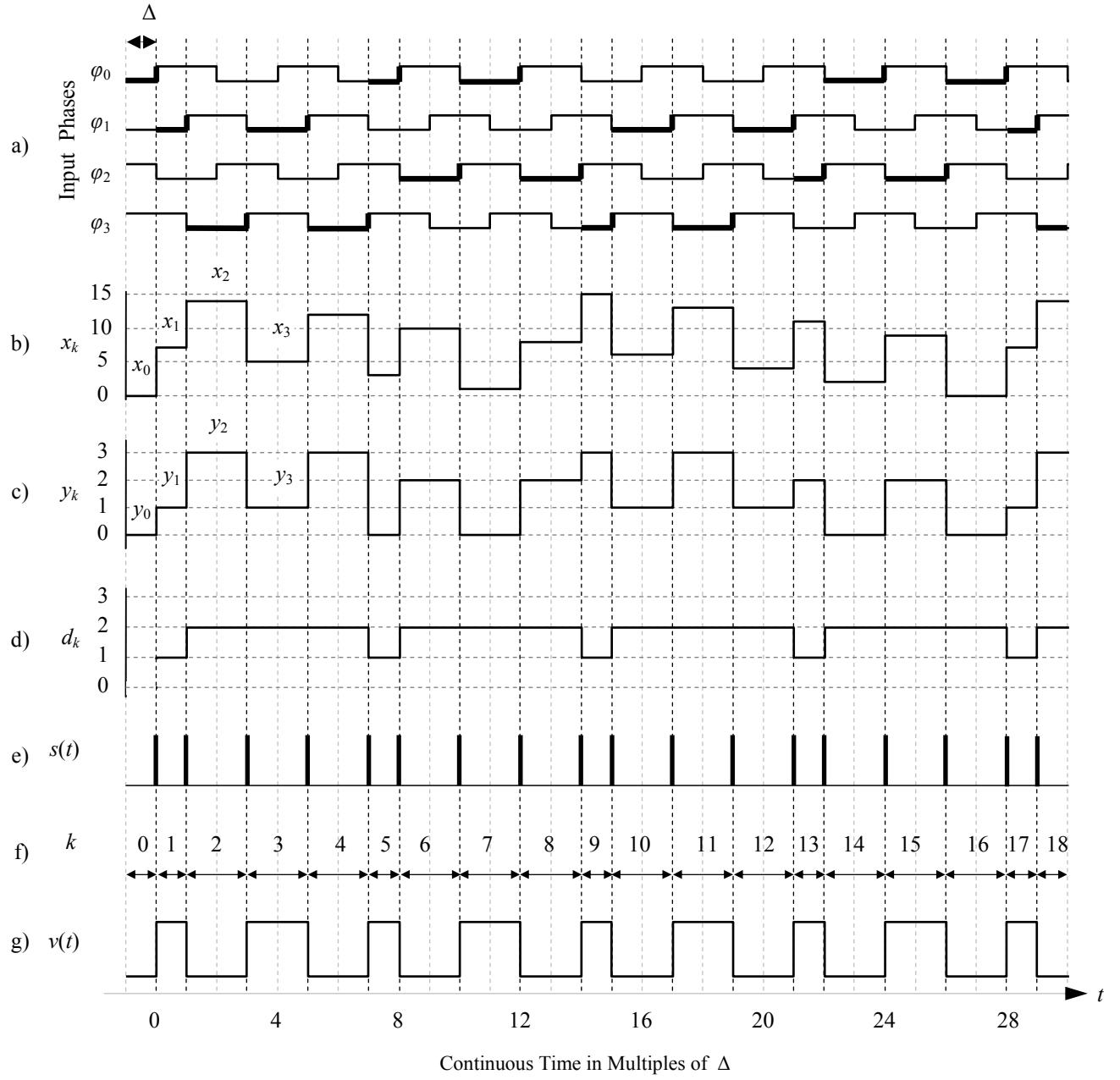


Figure 3 : The signals of the Flying-Adder synthesizer for the case where  $n = 4$ ,  $m = 2$  and  $w = 7$ .

$$f_{ave} = \begin{cases} \frac{2^{n-1}}{2^n - (2^m - 1)w} f_{clk} & \text{if } 0 < w < 2^{n-m} \\ \frac{2^{n-1}}{w} f_{clk} & \text{if } 2^{n-m} \leq w < 2^n \end{cases}$$

where  $f_{clk} = 1/T = 1/(2^m \Delta)$ .

#### B. Example of Sequences and Periods

For the case of  $n = 4$ ,  $m = 2$  and  $w = 7$ , see also the graphs in Figure 3, the discrete-time period is  $L = 4$ .

Starting with initial value  $x_0 = 0$  we calculate,  $x_1 = 7$ ,  $x_2 = 14$ ,  $x_3 = 5$ ,  $x_4 = 12$ ,  $x_5 = 3$ ,  $x_6 = 10$ , ..., using  $x_{k+1} = (x_k + w) \bmod 2^4$ . From these values we derive  $y_0 = 0$  we calculate,  $y_1 = 1$ ,  $y_2 = 3$ ,  $y_3 = 1$ ,  $y_4 = 3$ ,  $y_5 = 0$ ,  $y_6 = 2$  and so on, using equation  $y_k = x_k \bmod 2^2$ . Then we derive sequence  $d_1 = 1$ ,  $d_2 = 2$ ,  $d_3 = 2$ ,  $d_4 = 2$ ,  $d_5 = 1$ ,  $d_6 = 2$ , ... using  $d_k = (y_k - y_{k-1}) \bmod 2^2$ . Note that since  $d_k > 0$  it is  $d_k = \delta_k$  as well.

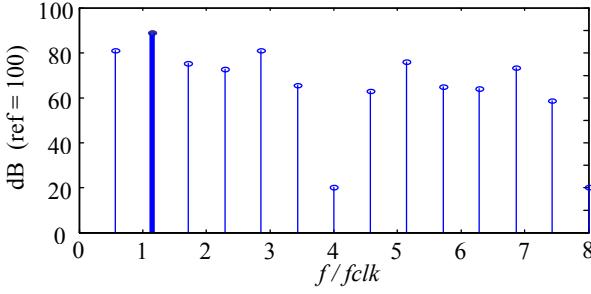


Figure 4 : The spectrum of  $v(t)$  when  $n = 4, m = 2, w = 7$   
The bold line corresponds to  $f_{ave}/f_{clk}$

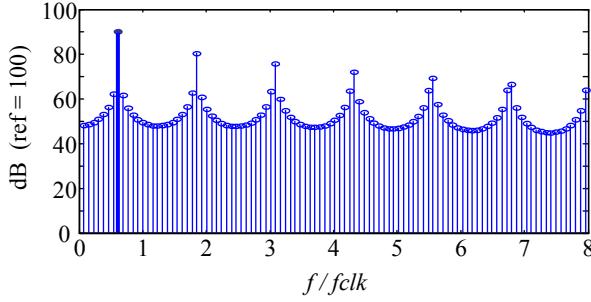


Figure 5 : The spectrum of  $v(t)$  when  $n = 8, m = 4, w = 207$   
The bold line corresponds to  $f_{ave}/f_{clk}$

The continuous-time period of the output of the MUX,  $s(t)$ , is  $T_P = \Delta(\delta_1 + \delta_2 + \delta_3 + \delta_4)$ , i.e.  $T_P = 7\Delta$ . Finally, the period of the output signal is  $T_v = \Delta \cdot 7 \cdot 4 / 2^4 = 7\Delta / 4$  which agrees with Theorem 3.

### C. Output Spectral Characteristics

Note that since the (fundamental) period of the output signal is  $T_v$ , the spectrum of  $v(t)$  consists of harmonics at frequencies  $f_\ell = \ell / T_v$ ,  $\ell = 0, 1, 2, \dots$ , and  $v(t)$  can be expressed as a Fourier series. Since  $T_v$  is known, the discrete spectrum can be easily derived numerically.

### D. Example of Output Spectrum

The spectrum of  $v(t)$  is presented in Figures 4 & 5 for two combinations of values of  $n, m$  and  $w$ .

### DETERMINISTIC JITTER

As we have seen in the above analysis and graphs, the output signal  $v(t)$  of the Flying-Adder is (in most cases) irregular in the sense that its period is composed of a number of pulses of unequal lengths. The following lemma provides an insight in the deterministic jitter of  $v(t)$ .

Lemma 1: If  $2^{n-m} \leq w < 2^n$  then the length of every 0 or 1 interval in  $v(t)$  is equal to one of the two values

$$\left[ \frac{w}{2^{n-m}} \right] \cdot \Delta, \quad \left[ \frac{w}{2^{n-m}} \right] \cdot \Delta + \Delta$$

Moreover, within every continuous-time period  $T_v$  of  $v(t)$ , there are at least two intervals (0 or 1 or both) of  $v(t)$  of lengths equal to each of the above values. Also, the length of every cycle (0-interval followed by 1-interval, or, vice versa) can take one of the two values

$$\left[ \frac{2w}{2^{n-m}} \right] \cdot \Delta, \quad \left[ \frac{2w}{2^{n-m}} \right] \cdot \Delta + \Delta$$

and within every continuous-time period  $T_v$  of  $v(t)$  there is at least one cycle of length equal to each of these two values.

The lemma allows us to determine upper bounds of pulse-to-pulse jitter. Moreover, it allows to bound the fractional jitter (jitter / period of signal) using the expression for  $f_{ave}$ . The next lemma provides the long-term deterministic jitter.

Lemma 2: Let  $2^{n-m} \leq w < 2^n$  and let  $\tilde{t}_j$  and  $\tilde{\tau}_j$  be the times of the j-th rising and falling edges respectively of a 50% duty-cycle periodic square wave of period equal to  $f_{ave}$  and zero initial phase, and, let  $t_j$  and  $\tau_j$  be the times of the j-th rising and falling edges respectively of  $v(t)$ . Then,

$$\tilde{t}_j - \Delta < t_j \leq \tilde{t}_j \quad \text{and} \quad \tilde{\tau}_j - \Delta < \tau_j \leq \tilde{\tau}_j.$$

### CONCLUSIONS

The Flying-Adder frequency synthesizer has been studied mathematically and its deterministic jitter has been estimated. Moreover, its average frequency and its spectrum have also been derived. The analytical results have been verified using MATLAB.

### REFERENCES

- [1] L. Xiu, "Some open issues associated with the new type of component: digital-to-frequency converter [Open Column]", *IEEE Circuits and Systems Magazine*, Vol. 8, Is. 3, 3<sup>rd</sup> Quarter 2008, pp:90-94.
- [2] H. Mair, L. Xiu, "An architecture of high-performance frequency and phase synthesis", *IEEE Journal of Solid-State Circuits* Vol. 35, Is. 6, Jun. 2000, pp.:835-846.
- [3] U. L. Rohde, "Microwave and Wireless Synthesizers: Theory and Design", first edition, Wiley-Interscience, 1997.
- [4] V. Manassewitsch, "Frequency Synthesizers", third edition, John Wiley & Sons, 1987.
- [5] W. F. Egan, Frequency Synthesis by Phase Lock. 2nd ed. New York: Wiley, 1999.
- [6] L. Xiu, "The concept of time-average-frequency and mathematical analysis of flying-adder frequency synthesis architecture", *IEEE Circuits & Systems Magazine*, Vol. 8, Is. 3, 3<sup>rd</sup> Quarter 2008.