Multiphase EXOR Frequency Mixers

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***Abstract*— Digital frequency mixing using EXOR gates requires minimal design effort and circuitry, and it can be easily included in any standard-CMOS integrated circuit. It suffers, however, from an output spectrum highly populated by strong spurious products. This work introduces the concept of multiphase EXOR mixing as an approach to reduce spurious content while maintaining design simplicity. Theory is supported by measurements using an FPGA implementation.**

***Index Terms*— Frequency Mixer, Frequency Synthesis, Multi-Phase, Spurious Signals, Spurious Free Dynamic Range**

I. Introduction

Frequency mixing (heterodyning) is very often used in multi-loop frequency synthesizers to achieve the desirable output frequency resolution, center frequency and frequency range, as well as in instrumentation equipment and RF and microwave circuit applications [1]-[2].

Ideally a frequency mixer produces frequency $\omega_1 + \omega_2$, or, $\omega_1 - \omega_2$, from frequencies ω_1 and ω_2. In reality, however, unwanted spurious products at some of the frequencies $k\omega_1 \pm m\omega_2$, $k, m = 1, 2, 3, \ldots$ are usually present, polluting the frequency spectrum [3].

It is noted that frequency mixing in synthesizers (as well as in many instrumentation and other equipment) is different from frequency mixing in RF receivers. In the first case both mixed signals have large amplitudes resulting in strong unwanted intermodulation products if we don't pay the necessary attention to the design; at the same time, noise is usually not an issue.

On the other hand, in RF receivers, the local oscillator's signal is strong but the input signal is typically very weak; so usually the unwanted intermodulation products are due only to the harmonics of the local oscillator, which are far away from the frequency range of interest (with the exception of Ultra Wide Band systems); moreover, the mixer's noise is a major issue.

Among the several types of mixers, the "pure" multiplier in Figure 1 is usually consider as the ideal one. The multiplier is typically implemented as a Gilbert cell with appropriate input pre-distortion [4], or using (MOS)FET devices with square-law characteristics. The filter at the output selects the desirable frequency component.

At the high end of the microwave frequency spectrum, the mixer type of Figure 2 is more popular, where the nonlinear element is realized by one or more diodes.

This general class includes the popular double-balanced diode mixer and the (passive) double-balanced FET mixer

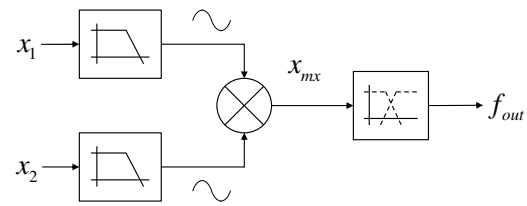

Fig. 1. The multiplier as a mixer

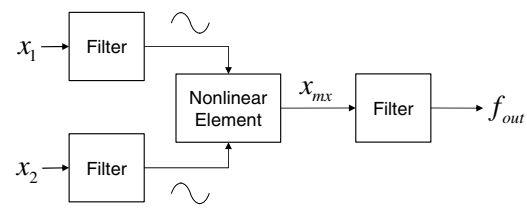

Fig. 2. Mixer using nonlinear element(s)

[3], [5]. When the amplitude of one of the signals (e.g., LO in receivers) is significantly larger than the turn-on voltage of the diodes (or FETs), and the amplitude of other one (e.g., RF signal) is much lower than that, both of these mixers can be viewed as (balanced) switches, conceptually leading to the general class of mixers in Figure 3 - shown as single-ended for simplicity.

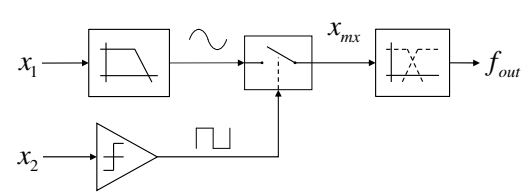

Fig. 3. Switching (or sampling) mixer

This mixer is essentially a switch multiplying a sine-wave with a square-wave leading to an output signal with frequency components at

$$w_m^{\pm} = \left| w_1 \pm (2m+1) w_2 \right| , \quad m = 0, 1, 2, \ldots$$

The switching mixer takes a sinusoidal and a digital signal as inputs. Assuming further that both signals are digital we come up with the class of mixers in Figure 4.

Although one can consider a whole family of combinational and sequential logic used in the mixer, the EXOR

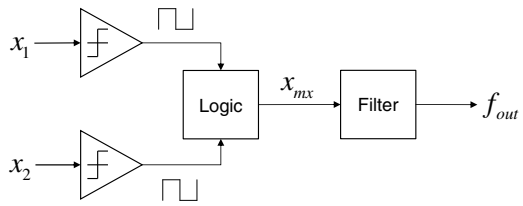

Fig. 4. Mixing of two square signals

gate is probably the most convenient option leading to the EXOR mixing operation

$$x_{mx}(t) = \hat{x}_1(t) \oplus \hat{x}_2(t)$$

where $\hat{x}_1$ and $\hat{x}_2$ are the 0-1 digital versions of the signals after the comparators, and $\oplus$ denotes the EXOR operation.

The EXOR gate has been used widely as a phase detector in PLLs and other circuits [1]. Phase detection corresponds to the special case here where $\omega_1 = \omega_2$.

As shown in Section II, EXOR is a form of multiplication and so it can be used as mixer. Being an extremely simple mixer driven directly by digital signals is frequently very convenient in frequency synthesis; e.g., when the signals come from digital dividers in PLLs, etc.

These properties of the EXOR mixer make it very attractive. However, since the square-waves $\hat{x}_1$ and $\hat{x}_2$ have all their odd harmonics, the output of the EXOR gate contains frequency components at

$$\omega^{\pm}_{(m,r)} = \big|(2m+1)\omega_1 \pm (2r+1)\omega_2\big| \tag{1}$$

where $m, r = 0, 1, 2, \ldots$.

Multi-phase mixing is considered in this work as a way to reduce the spurious content at the output of EXOR-based mixers while maintaining their hardware simplicity.

This approach is also supported by the fact that balanced, and moreover doubly-balanced, mixers have cleaner output signals since balancing suppresses even order frequency components. Quadrature and distributed mixers extend this concept one step further [5].

In a sense, a multi-phase EXOR mixer can be considered as the quadrature doubly-balanced, version of a single-diode mixer, or as the digital version of a distributed mixer.

II. Signal Representations

A periodic square-waveform ± 1V signal of frequency ω_1 can be expressed as the Fourier series

$$a(t) = \frac{2}{\pi i} \sum_{k\,:\,\text{odd}} \frac{1}{k}\, e^{i\,k\,\omega_1\,t}. \tag{2}$$

Since $a(t) \in \{-1, +1\}$ for every t, the corresponding 0-1 (V) digital signal $A(t)$ is given by

$$A(t) = \frac{1}{2} + \frac{1}{2} a(t). \tag{3}$$

$X \setminus Y$	0	1
0	0	1
1	1	0

TABLE I
EXOR Truth table

Similarly, let $b(t)$ be the periodic square-waveform ± 1V signal of frequency ω_2,

$$b(t) = \frac{2}{\pi i} \sum_{m\,:\,\text{odd}} \frac{1}{m}\, e^{i\,m\,\omega_2\,t}. \tag{4}$$

and $B(t) = \frac{1}{2} + \frac{1}{2} b(t)$ be the corresponding digital one.

Consider now the standard EXOR operation in Boolean algebra, denoted as $\oplus$. Its truth table is shown in Table I where X and Y are Boolean variables.

Now suppose that X and Y can take the values 0 and 1 but this time in the algebra of real numbers, $\mathbb{R}$. Then, function EXOR can be expressed in $\{0, 1\} \subset \mathbb{R}$ as

$$X \oplus Y = X + Y - 2XY. \tag{5}$$

We see from (5) that EXOR is in some sense the (real) multiplication of X and Y, plus a copy of them.

Since multiplication is a good approach to frequency mixing, one can consider using the EXOR gate for doing so. Note, however, that (5) is valid only when $X, Y \in \{0, 1\} \subset \mathbb{R}$ and so the EXOR gate works as a multiplier only for square-wave (digital) signals.

III. EXOR Mixing

The EXOR operation between the digital signals $A(t)$ and $B(t)$ can be expresses as

$$A(t) \oplus B(t) = \frac{1}{2} - \frac{1}{2} a(t) b(t) \tag{6}$$

where we have used expression (3) and the corresponding one for $B(t)$. Replacing (2) and (4) in (6) gives

$$A(t) \oplus B(t) = \frac{1}{2} + \frac{2}{\pi^2} \sum_{k,m\,:\,\text{odd}} \frac{1}{k\,m}\, e^{i\,(k\,\omega_1 + m\,\omega_2)\,t}$$

which after some algebraic manipulation can be written as

$$A(t) \oplus B(t) = \frac{1}{2} + \frac{4}{\pi^2} \sum_{\substack{k,m>0 \\ k,m\,:\,\text{odd}}} \frac{\cos\big((k\,\omega_1 + m\,\omega_2)\,t\big)}{k\,m} - \frac{4}{\pi^2} \sum_{\substack{k,m>0 \\ k,m\,:\,\text{odd}}} \frac{\cos\big((k\,\omega_1 - m\,\omega_2)\,t\big)}{k\,m}. \tag{7}$$

Therefore, if no frequencies $k\,\omega_1 \pm m\,\omega_2$ overlap[1] for different pairs of positive odd integers (k, m), the spectrum of $A(t) \oplus B(t)$ is composed of frequency components at

$$\omega^{\pm}_{(r_1,r_2)} = \big|(2r_1+1)\omega_1 \pm (2r_2+1)\omega_2\big| \tag{8}$$

[1]If some frequencies $k\,\omega_1 \pm m\,\omega_2$ overlap, the amplitudes $A_{(r_1,r_2)}$ need to be modified appropriately

with amplitudes

$$A^{\pm}_{(r_1,r_2)} = \frac{4}{\pi^2(2r_1+1)(2r_2+1)} \tag{9}$$

where $r_1, r_2 = 0, 1, 2, \ldots$.

Usually the desirable frequency components are $\omega_1 \pm \omega_2$. The spectrum of the EXOR mixer, however, given by (8) is quite "rich". The first few frequency terms are shown in Figure 6 where "+" and "Δ" correspond to $\cos\big((k\,\omega_1 + m\,\omega_2)t\big)$ and $\cos\big((k\,\omega_1 - m\,\omega_2)t\big)$ frequency terms respectively in expression (7).

IV. Multi-Phase EXOR Mixer

One way to reduce the EXOR mixer's spurs is using multiple phases of the periodic signals $A(t)$ and $B(t)$, EXOR the corresponding phases and average, as shown in Figure 5.

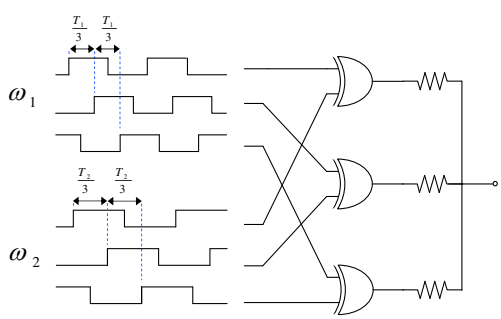

Fig. 5. Multi-Phase EXOR Mixer (shown for 3 phases)

We consider the EXOR mixing of phase-shifted replicas of signals $A(t)$ and $B(t)$ where the phase offsets are of the form

$$\phi_n = \frac{2\pi}{N}\,n \quad \text{and} \quad \psi_n = \frac{2\pi}{N}\,n \tag{10}$$

and $n = 0, 1, 2, \ldots, N-1$. This choice is appropriate when we want $|\omega_1 - \omega_2|$ as the output frequency.

From (2) and (4) the phase-shifted copies of the digital waveforms are expressed as

$$a(t,\phi_n) = \frac{2}{\pi i}\sum_{k\,:\,\text{odd}} \frac{1}{k}\, e^{ik(\omega_1 t + \frac{2\pi}{N}n)} \tag{11}$$

and

$$b(t,\psi_n) = \frac{2}{\pi i}\sum_{m\,:\,\text{odd}} \frac{1}{m}\, e^{im(\omega_2 t + \frac{2\pi}{N}n)} \tag{12}$$

respectively. In addition we set $A(t,\phi_n) = \frac{1}{2} + \frac{1}{2}a(t,\phi_n)$ and $B(t,\psi_n) = \frac{1}{2} + \frac{1}{2}b(t,\psi_n)$ which, through (6) give

$$\begin{aligned} A(t,\phi_n) \oplus B(t,\psi_n) &= \frac{1}{2} - \frac{1}{2}a(t,\phi_n)b(t,\psi_n) \\ &= \frac{1}{2} + \frac{2}{\pi^2}\sum_{k,m:\,\text{odd}} \frac{e^{i(k\omega_1+m\omega_2)t + i\frac{2\pi}{N}(k+m)n}}{km} \end{aligned}$$

Summation of $A(t,\phi_n) \oplus B(t,\psi_n)$ over all phases gives

$$\sum_{n=0}^{N-1} A(t,\phi_n) \oplus B(t,\psi_n) = \frac{N}{2} + \frac{2}{\pi^2}\sum_{k,m:\,\text{odd}} \frac{q(k,m)}{km} e^{i(k\omega_1+m\omega_2)t}$$

where functions q is defined as

$$q(k,m) = \sum_{n=0}^{N-1} e^{i\frac{2\pi}{N}(k+m)n} = \begin{cases} N & \text{if } k+m \in N\mathbb{Z} \\ 0 & \text{otherwise} \end{cases} \tag{13}$$

and $N\mathbb{Z} = \{rN | r \in \mathbb{Z}\}$. Combining the two equations above we derive

$$\begin{aligned} \sum_{n=0}^{N-1} A(t,\phi_n) \oplus B(t,\psi_n) &= \frac{N}{2} + \frac{2N}{\pi^2}\sum_{\substack{k,m\,:\,\text{odd} \\ k+m\in N\mathbb{Z}}} \frac{e^{i(k\omega_1+m\omega_2)t}}{km} \\ &= \frac{N}{2} + \frac{4N}{\pi^2}\sum_{\substack{k,m\,:\,\text{odd} \\ k,m>0 \\ k+m\in N\mathbb{Z}}} \frac{\cos\big((k\omega_1+m\omega_2)t\big)}{km} \\ &\quad - \frac{4N}{\pi^2}\sum_{\substack{k,m\,:\,\text{odd} \\ k,m>0 \\ k-m\in N\mathbb{Z}}} \frac{\cos\big((k\omega_1-m\omega_2)t\big)}{km} \end{aligned}$$

Assuming that no frequencies $k\,\omega_1 \pm m\,\omega_2$ overlap[2] for different feasible pairs (k,m), the spectrum of the N-phase EXOR mixer is composed of the signal components at

$$\omega^{+}_{(k,m)} = k\omega_1 + m\omega_2, \tag{14}$$

where k, m are odd and positive with $k + m \in N\mathbb{Z}$, and,

$$\omega^{-}_{(k,m)} = \big|k\omega_1 - m\omega_2\big| \tag{15}$$

where here k, m are odd and positive such that $k - m \in N\mathbb{Z}$. In both cases the corresponding amplitudes are

$$A^{\pm}_{(k,m)} = \frac{4N}{\pi^2 km}. \tag{16}$$

Comparing (14) and (15) with (8) we see that, in theory, the spectrum of the N-phase EXOR mixer contains significantly less spurs, especially for large N.

The first few frequencies given by (14) and (15) are indicated in Figures 6-8, for $N = 1, 2, 3$ and 7, with "+" and "Δ" corresponding to $\omega^{+}_{(k,m)}$ and $\omega^{-}_{(k,m)}$, respectively.

In theory, two phases, $N = 2$ have no advantage versus one, $N = 1$ because the sum $k + m$ and difference $k - m$ of any two odd numbers k, m is even. For similar reason, $N = 7$ results in significantly cleaner spectrum.

V. Measurements and Applications

Ring oscillators naturally produce the multi-phase digital signals needed for multi-phase EXOR mixing and so PLLs using ring oscillators form a convenient application framework. In the design attention should be paid to having accurate phase offsets in the signals.

[2]If they do, the amplitudes $A_{(k,m)}$ need to be modified appropriately.

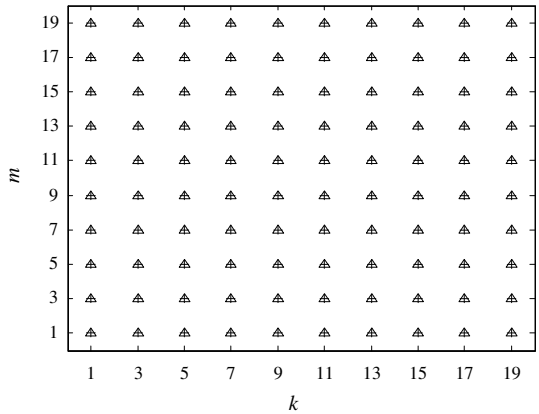

Fig. 6. First few output signal component frequencies, $N = 1, 2$

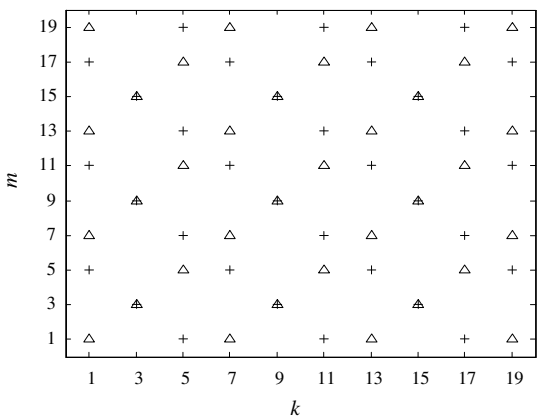

Fig. 7. First few output signal component frequencies, $N = 3$

Alternatively, the N phases can be derived by feeding two dividers-by-N with frequencies $N\omega_1$ and $N\omega_2$. Shift registers are perhaps the most convenient option.

Two shift registers and a 7-phase EXOR mixer were implemented in an FPGA. The frequencies, after the shift registers, were $f_1 = 3.5714$MHz and $f2 = (21/13)f_1 = 5.7692$MHz[3] The desirable output frequency is $f_2 - f_1 = 2.1978$MHz.

The output spectrum from 0 to 6.5MHz is shown in Figure 9. Within this range, the Spurious Free Dynamic Range (SFDR) is 49dB, which is a 40dB improvement compared to the 9dB SFDR measured using a single-EXOR mixer.

Only spurs of the type $\omega^{-}_{(k,m)}$, given by expression (15), are present within the frequency range shown. The pairs (k, m) indicate the dominant spurs, and the numbers above (in dB) show the reduction of their amplitude with respect to those of the single-EXOR mixer.

[3]Overlapping of spurs occurs here but has minor effect on the estimated spectrum since the overlapping spurs have significant magnitude difference.

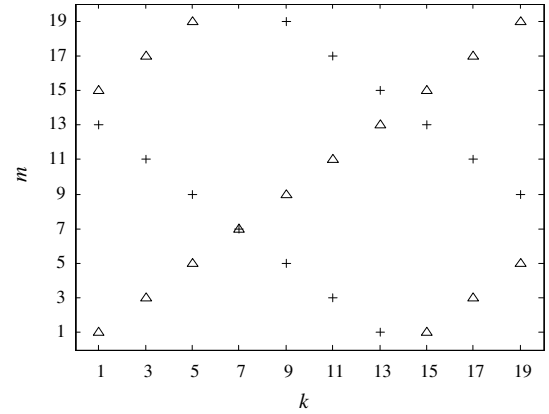

Fig. 8. First few output signal component frequencies, $N = 7$

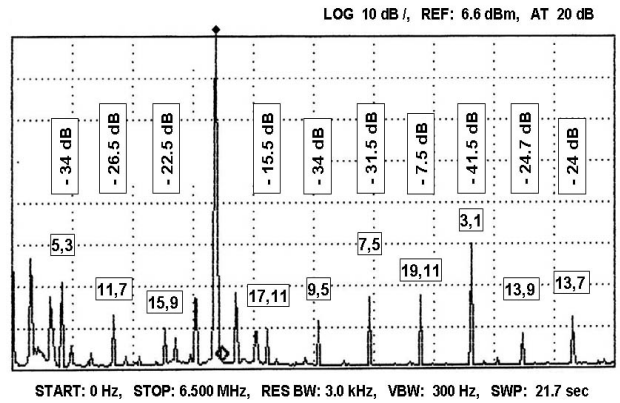

Fig. 9. The output spectrum of a 7-phase EXOR mixer when $f_1 = 3.5714$MHz and $f2 = 5.7692$MHz, $(f_2 = 21/13f_1)$. The desirable output frequency is $f_2 - f_1 = 2.1978$MHz.

VI. Conclusions

Multi-phase EXOR mixers were discussed in the framework of frequency synthesis. It was shown that they can have a significantly cleaner spectrum than the simple EXOR mixer while maintaining circuit simplicity. Multi-phase EXOR mixers are essentially digital circuits except the averaging output network that can be realized using resistors or capacitors.

References

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