

Nanoscale Data Storage Devices Capacity and Encoding Schemes

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Abstract: Crossbar switching networks formed by nanowire arrays and ohmic switches are promising future data storage devices. This paper presents the information capacity limits of these devices as well as simple encoding schemes that can be used to store and recover the data. It is shown that for large array sizes N, M , capacity asymptotically approaches $(N+M)\log_2(N+M)$ bits. The proposed encoding scheme allows for using $N\log_2(M)$ bits ($N \geq M$) using low complexity combinatorial circuitry.

1. INTRODUCTION

Recent advances in nanotechnology have enabled the development of crossbar switching networks (CSNs) using nanowires [1]-[7]. The small size and high density of these structures makes them favorable candidates for future high density interconnect, computation and information storage devices [1]-[22]. Here we consider rectangular $N \times M$ crossbar switching networks with *ohmic* (contact) switches between every horizontal and every vertical wire as shown in Figure 1. Every switch has two possible states, one of high and one of low resistance (*open* and *closed* respectively). We refer to this class of crossbar switching networks as *R-CSNs* to distinguish them from crossbar switching networks with *semiconductive* (diode) switches, *D-CSNs*, shown in Figure 2.

The exact information storage capacity of $N \times M$ *R-CSNs* is derived. Simple approximate expressions are given as well. Additionally, it is shown that for $N \rightarrow \infty$ the capacity of square $N \times N$ *R-CSNs* approaches $2N\log_2 N$ bits and that for $N, M \rightarrow \infty$ and $N/M \rightarrow a > 0$, the capacity of $N \times M$ *R-CSNs* approaches $(N+M)\log_2(N+M)$. This is in contrast to the capacity of $N \times N$ *D-CSNs* (Figure 2) which is N^2 bits. Although *D-CSNs* are superior to *R-CSNs* in terms of capacity, *R-CSNs* are more prevalent nano-structures currently and are expected to have significant applications in the future [1]-[2].

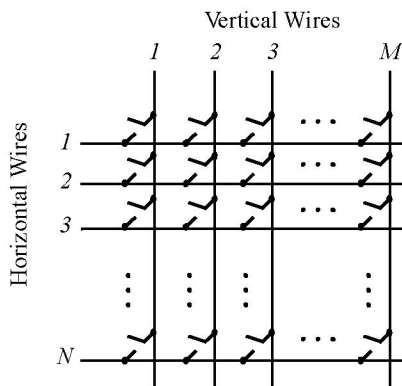


Figure 1: $N \times M$ *R-CSN*

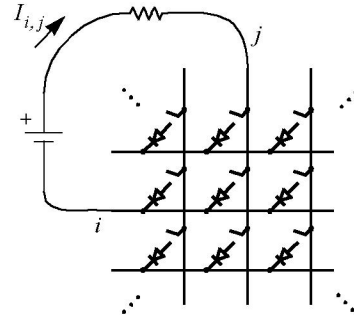


Figure 2: *D-CSN* and the extraction of its configuration.

2. PRELIMINARIES

Throughout the paper we consider rectangular $N \times M$ CSNs with N horizontal and M vertical wires. The numbering of the wires is as shown in Figure 1, i.e. similar to that of an $N \times M$ matrix. The pair (i, j) , with $i = 1, 2, \dots, N$ and $j = 1, 2, \dots, M$, is used to denote either the pair of the i^{th} horizontal and j^{th} vertical wires, or, the corresponding switch between them. For pictorial simplification in *R-CSNs only*, a black dot at the intersection (i, j) is used to indicate that switch (i, j) is closed as shown in Figure 3.

We say that two wires (horizontal, vertical or both) are *connected* if there is a path of closed switches between them, otherwise, they are *disconnected*.

To estimate the information capacity of *R-CSNs* we make the following intuitive assumptions: 1) The switches have high open/closed resistance ratio $r_{\text{open}}/r_{\text{closed}}$; 2) The resistance of the wires is much smaller than the r_{open} of the switches; 3) The states of the switches, *closed* or *open*, can be set independently. These assumptions imply that by measuring the resistance between any two wires we can accurately conclude whether they are *connected* or not; the resistance measurement provides the correct *binary* answer.

The way the switches of a CSN are setup (closed/open) is called a *configuration* of the CSN. According to our assumptions, both $N \times M$ *R-CSN* and *D-CSN* have $2^{N \times M}$ possible configurations.

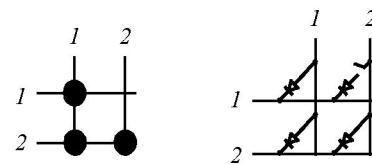


Figure 3: An *R-CSN*, a *D-CSN* having the same configuration

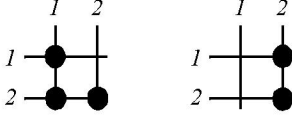


Figure 4: Two distinguishable configurations of a 2×2 R-CSN

3. R-CSN VS. D-CSN; THE MAIN DIFFERENCE

Consider a D -CSN, like that in Figure 2, (i.e. with diode switches). Let the $N \times M$ switches have a given, yet *unrevealed to us*, configuration. To extract the configuration we can perform a series of $N \times M$ measurements; specifically: for $i = 1, 2, \dots, N$ and $j = 1, 2, \dots, M$ we measure the current $I_{i,j}$ as in Figure 2. If $I_{i,j}$ is non-negligible we conclude that the switch (i,j) is closed, otherwise we conclude that the switch (i,j) is open. The main issue here is that the $N \times M$ connections (not switches) are *independent* in the sense that for every pair (i,j) , current $I_{i,j}$ depends *only* on the state of switch (i,j) . This is in contrast to the case of R-CSNs. In Figure 4 for example the resistance between the 1^{st} horizontal and the 2^{nd} vertical wires is low in both configurations; concluding that the $(1,2)$ switch is closed in both cases is incorrect. Therefore, in R-CSNs, the measured currents do not necessarily represent the states of the corresponding switches.

Although we cannot tell which of the switches are closed or open in the left configuration of Figure 4, we can still distinguish it from the right one (in Figure 4) by examining the whole set of $6 = \binom{2+2}{2}$ resistance measurements (i.e. for every pair of the four wires). By doing so we infer that the pairs of *connected wires* in the left configuration are $(1,1)$, $(1,2)$, $(2,1)$ and $(2,2)$ while the pairs of connected wires in the right configuration are $(1,2)$ and $(2,2)$. However, we cannot distinguish between the two configurations of the switches in Figure 5. They result in the same electrical behavior of the network, i.e. all wires are connected together. The above examples and discussion are extended directly to R-CSNs of any sizes N, M .

It is reasonable to assume that the R-CSN is connected to the rest of the circuitry through its terminals and that the states of the switches cannot be observed from outside or be extracted by any other way, i.e. the switches are *hidden* inside the CSN.

An $N \times M$ R-CSN along with a given configuration of its switches is called an $(N+M)$ -terminal *device*. A device corresponds to the whole set of information we can extract by measuring the resistance between all pairs of R-CSN's terminals (wires), i.e. $\binom{N+M}{2}$

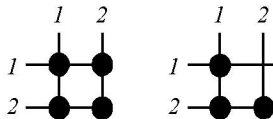


Figure 5: Two different but indistinguishable configurations of a 2×2 R-CSN

measurements. The device captures the electrical behavior of the R-CSN, with a particular switches' configuration.

For example, it is easily verifiable that the 16 configurations (set-ups of the switches) of a 2×2 R-CSN realize only 12 devices.

3.1 The Notion of Capacity for R-CSNs

Consider an $N \times M$ D -CSN like that in Figure 2. All 2^{NM} configurations of the switches are distinguishable using the current measurement tests. Every configuration corresponds in a one-to-one fashion to a unique electrical behavior i.e. a *device*, and the 2^{NM} distinct electrical behaviors are 2^{NM} possible states of the D -CSN. Therefore, the information capacity of the $N \times M$ D -CSN is $NM = \log_2 2^{NM}$ bits. By extending this concept to R-CSNs, the storage capacity of $N \times M$ R-CSNs is $B = \log_2 D$ bits, where $D = D(N, M)$ is the number of devices $N \times M$ R-CSNs realize.

4. EXACT EXPRESSIONS FOR R-CSNs' CAPACITY

The following theorem provides an expression of the exact information capacity of $N \times M$ R-CSNs. The expression involves the Stirling numbers of the second kind $S(n, m)$ [23], given by (1).

$$S(n, m) = \frac{1}{m!} \sum_{t=0}^m (-1)^{m-t} \binom{m}{t} t^n \quad (1)$$

Theorem 1: The information capacity of $N \times M$ R-CSNs is $B = \log_2(D)$ bits where D is the number of realizable devices.

$$D = \sum_{q=0}^{\min\{N, M\}} S(N+1, q+1) \cdot S(M+1, q+1) \cdot q! \quad (2)$$

Figure 6 shows the graph of information capacity, $B(N, M)$, for N, M varying from 1 to 1024. The capacity is an increasing function of both variables and symmetric, $B(N, M) = B(M, N)$.

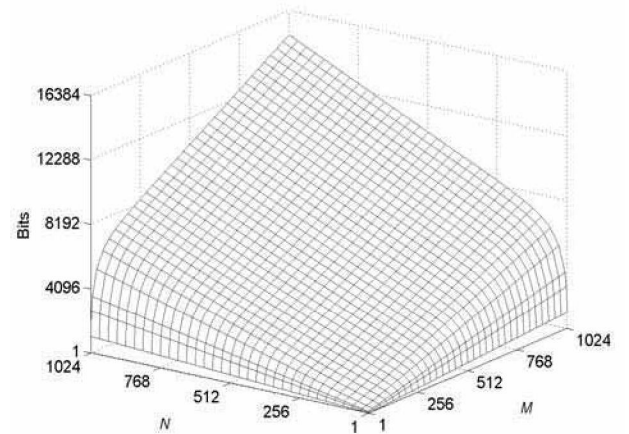


Figure 6: Information Capacity $B(N, M)$ of $N \times M$ R-CSNs

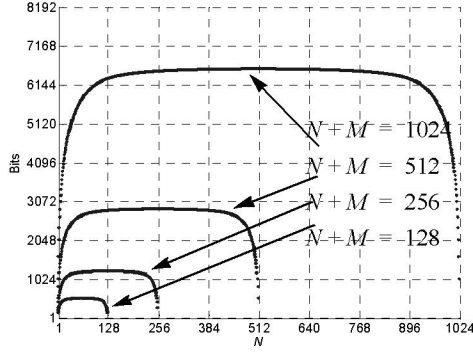


Figure 7: Information Capacity of $N \times M$ R-CSNs, as a function of N , when $N + M$ is fixed

Figure 7 shows the graph of the capacity $B(N, M)$ when $N + M$ is fixed. The capacity is maximized when $N = M$.

In the case of square, $N \times N$, R-CSNs, expression (2) becomes

$$D = \sum_{q=0}^N S(N+1, q+1)^2 \cdot q! \quad (3)$$

and the capacity $B(N)$ is shown in Figure 8 along with its lower and upper bounds $N \log_2(N)$ and $2N \log_2(N)$ respectively.

5. APPROXIMATE EXPRESSIONS FOR THE CAPACITY

The exact calculation of R-CSNs' capacity is not trivial for very large sizes M and N ; this motivates the derivation of low complexity approximate expressions, one of which is:

$$B_1(N, M) = (N + M - q) \cdot \log_2(q) + q \cdot \log_2(e) \quad (4)$$

with $q = \frac{N+M}{\ln(N+M)}$

Where, $\ln$ is the natural logarithm and $e = 2.7183\dots$. The approximation of $B(N, M)$ by $B_1(N, M)$ improves percentage-wise as $N, M \rightarrow \infty$. The approximation is improved further if q is replaced by the real solution of equation $N + M = \tilde{q} \cdot \log_2(\tilde{q})$.

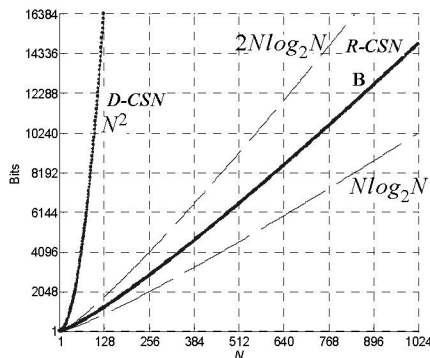


Figure 8: Exact expression and bounds for the capacity of $N \times N$ R-CSNs, in comparison with the capacity of D-CSNs

A simpler but less accurate approximation of $B(N, M)$ for large values of both N and M is given by (5).

$$B_2(N, M) = (N + M) \cdot \log_2\left(\frac{2}{3} \cdot \frac{N + M}{\ln(N + M)}\right) \quad (5)$$

For square $N \times N$ R-CSNs expressions (4) and (5) become

$$B_1(N) = (2N - q) \cdot \log_2(q) + q \cdot \log_2(e)$$

with $q = 2N / \ln(2N)$

and

$$B_2(N) = 2N \cdot \log_2\left(\frac{4N}{3 \ln(2N)}\right).$$

6. ASYMPTOTIC CAPACITY OF R-CSNs

The asymptotic behavior of $B(N, M)$ and $B(N)$ for large values of N and M is given by the following two theorems.

Theorem 2: The capacity $B(N, M) = \log_2 D(N, M)$ of $N \times M$ R-CSNs has the property

$$\lim_{\substack{N, M \rightarrow \infty \\ N/M \rightarrow a > 0}} \frac{B(N, M)}{(N + M) \cdot \log_2(N + M)} = 1 \quad (6)$$

Figure 9 shows the ratio of the capacity over the asymptotic expression $(N + M) \cdot \log_2(N + M)$. Due to the very large values of N and M , the approximate formula $B_1(N, M)$ was used instead of $B(N, M)$. The error introduced is insignificant.

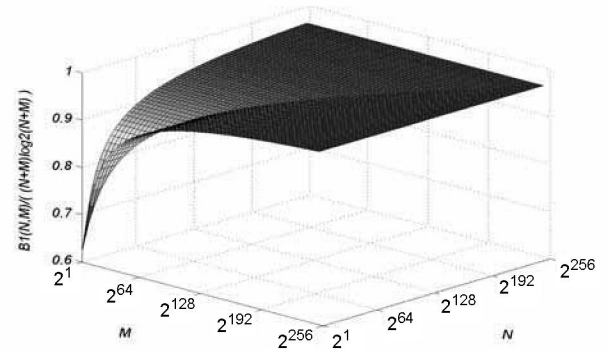


Figure 9: The capacity of $N \times M$ R-CSNs relative to $(N + M) \cdot \log_2(N + M)$ for very large values of N and M

Regarding the square $N \times N$ R-CSNs we have a similar result:

Theorem 3: The capacity $B(N) = \log_2 D(N)$ of $N \times N$ R-CSNs has the property

$$\lim_{N \rightarrow \infty} \frac{B(N)}{N \log_2(N)} = 2 \quad (7)$$

Note that this result is stronger than saying ' $B(N)$ is in the order of $N \log_2 N$ ', which can be derived using complexity arguments.

7. DATA ENCODING FOR R -CSNs

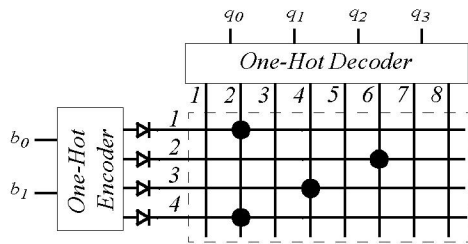


Figure 10: A paradigm of encoding in a 4×8 R -CSN

For R -CSNs to be practically useful for information storage, coding of binary data into configurations of their switches is required; and decoding to retrieve the information. Following the discussion in Section 3, it becomes clear that using the whole capacity of R -CSNs may result to very complicated coding schemes. To this end, sacrificing part of R -CSNs' capacity may allow for simpler coding.

Consider the case where $N = 2^n$, $M = 2^m$ for some n, m . Now suppose that *exactly one closed switch* is allowed in every row (horizontal wire). This results in M^N choices. Using the standard *one-hot* encoder and decoder, as illustrated in Figure 10, all M^N such configurations are distinguishable. So, using this encoding scheme, one can store $\log_2(M^N) = m2^n = N \log_2(M)$ bits of information. More examples and discussion on encoding and decoding schemes can be found in [2], [24]–[28].

8. CONCLUDING REMARKS

The information storage capacity of $N \times M$ crossbar switching network arrays with ohmic (contact) switches (R -CSNs) was derived explicitly. The capacity is asymptotic to $2N \log_2(N)$ for square $N \times N$ R -CSNs and asymptotic to $(N+M) \cdot \log_2(N+M)$ in general for large sizes N, M . A simple encoding scheme was discussed to illustrate the trade-off between encoding complexity and capacity usage.

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