Rapid Intermodulation Distortion Estimation in Fully Balanced Weakly Nonlinear Gm-C Filters using State-Space Modeling

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ABSTRACT

State-space modeling of fully differential *Gm-C* filters with weak nonlinearities is used to develop a fast algorithm for intermodulation distortion estimation. It results in simple analytic formulas that apply directly to *Gm-C* filters of *any order* and any fully balanced topology. The algorithm has been verified using SpectreS (SPICE) and Simulink simulation. Theory and simulation results are found in good agreement.

Categories and Subject Descriptors

B.7 [**Integrated Circuits**]:

General Terms

Algorithms, Design, Theory, Performance

Keywords

G_m-C filters, harmonic distortion, weak nonlinearity, circuit analysis, fully balanced, fully differential, distortion model, perturbation, state space, fast algorithm.

1. INTRODUCTION

Over the past few decades, continuous-time $G_m - C$ filters have become one of the most popular classes of active filters used in a vast variety of applications [1]. Significant efforts have also been devoted to analyzing and optimizing all aspects of their performance.

Some of the early $G_m - C$ filters were implemented in bipolar technology [2], [3]. However, the advances in CMOS semiconductor technologies provided an ideal ground for them [4]. Among the many types of circuit topologies for $G_m - C$ filters, the fully differential ones are almost always preferred due to their significantly higher linearity.

Although the methodology presented in this paper can be applied to a large variety of linear circuits, the focus is on distortion estimation in band-pass $G_m - C$ filters. Band-pass filters are important in many applications, including wireless communication systems, and their nonlinearity (along with noise) determines the performance of the whole system. By their nature, (narrow) band-pass filters suffer mainly from intermodulation distortion (IMD), and in most practical cases by third order IMD, (IM_3).

The standard test signal for intermodulation distortion estimation is the two-beat signal $u = k_1 \sin(2\pi f_1) + k_2 \sin(2\pi f_2)$. In band-pass filters f_1 and f_2 are chosen to be close to each other and within the pass-band. The dominant intermodulation products that usually appear within the pass-band are at frequencies $(m+1)f_1 - mf_2$ and $(m+1)f_2 - mf_1$, $m = 1, 2, \ldots$. Also, in weakly nonlinear filters the magnitude of the IMD products drops rapidly with their order. So, in practice, IMD analysis in band-pass filters focuses on the 3^{rd} order IMD component (IM_3) at frequency $2f_1 - f_2$ [5], [6].

The most popular measure of IM_3 is the ratio of the amplitude of the parasitic signal at frequency $2f_1 - f_2$ over the amplitude of the signal at f_1, i.e. IM_3 is defined relatively to the beat at f_1, [5], [7]. Other measures of IM_3 are the 3^{rd} order intercept point (IP_3), the 1-dB compression point (P_{1dB}), and the Spurious-Free Dynamic Range(SFDR), e.g. [5], [8], [9] and [10].

Intermodulation distortion estimation can be done at the transistor or the filter level. Basic transistor amplifying units can be treated as input-output static (memoryless) functions [6], [7], [11], [12], while filters are dynamical systems (they have memory) which makes their IMD estimation a more complicated problem. Volterra series has been the most popular tool to address it e.g. [13], [14], [15]. However, deriving analytical results is practically limited to low order filters, e.g. [16], [17]. In most cases the Volterra series method leads to complicated algebraic expressions. An alternative approach was introduced in [18] where the total IMD product of the filter was approximated by the sum of the IMD products of the individual transconductors linearly propagated to the output of the filter through the corresponding partial transfer functions.

In contrast to existing techniques this paper introduces a method for very fast IMD estimation that is based on *state-space* modeling and mathematical treatment of the filter [19]-[20]. The proposed method leads to analytic expressions that explicitly depend on the structural matrices of the filter and the components' values providing a simple and general tool for IMD estimation. The derived formulas are *valid for $G_m - C$ filters of any order and any fully differential topology.*

To evaluate the developed algorithm, a Tow-Thomas $G_m - C$ biquad was designed in a $0.5\mu m$ standard CMOS process using Cadence and was simulated in SpectreS (SPICE) and Simulink.

2. FULLY DIFFERENTIAL WEAKLY NONLINEAR TRANSCONDUCTORS

Fully differential transconductors are preferred when low distortion is required in linear circuits such as $G_m - C$ filters, [1], [3], [10], [4], [21]. Because of their balanced structure, they exhibit mainly *odd* order nonlinearity. Moreover, in most practical cases the third order nonlinear term is dominant and higher order terms can be safely ignored.

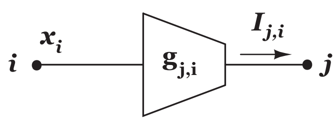

Figure 1: Transconductor model

Let $I_{j,i}$ be tranconductor's output current flowing into node j when its input is connected to node i as shown in Figure 1 (single-ended notation is used for notational convenience). Following the discussion above, $I_{j,i} = g_{j,i}x_i + e_{j,i}x_i^3$. Moreover, in many practical cases, $e_{j,i}$, is proportional to $g_{j,i}$ implying that

$$I_{j,i} = g_{j,i}\, x_i + \alpha\, g_{j,i}\, x_i^3 \tag{1}$$

These assumptions are typical in estimating the distortion of filters with fully differential transconductors [14], [18] and are adopted here.

The transconductance $g_{j,i}$ and the (small) constant α, which has units of Volt^{-2}, relate directly to the transistor-level design of the transconductor. They can be derived *analytically* or, *numerically* by fitting a third order polynomial to the $I - V$ characteristic of the transconductor.

3. STATE-SPACE MODEL OF $G_M - C$ FILTERS WITH WEAKLY NONLINEAR FULLY DIFFERENTIAL TRANSCONDUCTORS

The state-space model is introduced using the following example. Consider the second order $G_m - C$ filter in Figure 2.

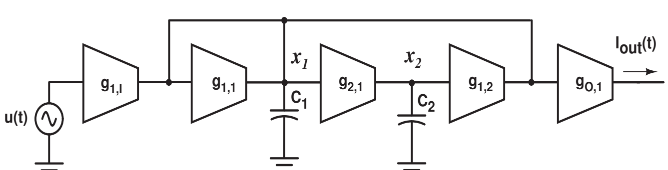

Figure 2: Second order bandpass $G_m - C$ filter

Using equation (1) the state-space formulation of the filter is given by the following system of differential equations

$$\underbrace{\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix}}_{\dot{\mathbf{x}}} = \underbrace{\begin{bmatrix}\frac{g_{1,1}}{C_1} & \frac{g_{1,2}}{C_1}\\ \frac{g_{2,1}}{C_2} & 0\end{bmatrix}}_{\mathbf{A}}\underbrace{\begin{bmatrix}x_1\\ x_2\end{bmatrix}}_{\mathbf{x}} + \alpha \underbrace{\begin{bmatrix}\frac{g_{1,1}}{C_1} & \frac{g_{1,2}}{C_1}\\ \frac{g_{2,1}}{C_2} & 0\end{bmatrix}}_{\alpha\mathbf{A}}\underbrace{\begin{bmatrix}x_1^3\\ x_2^3\end{bmatrix}}_{\mathbf{x}^{\bullet 3}} + \underbrace{\begin{bmatrix}\frac{g_{1,I}}{C_1}\\ 0\end{bmatrix}u}_{\mathbf{b}u} + \underbrace{\alpha \begin{bmatrix}\frac{g_{1,I}}{C_1}\\ 0\end{bmatrix}u^3}_{\alpha\mathbf{b}u^3}$$

and the output algebraic equation ($y = I_{out}$)

$$y = \underbrace{\begin{bmatrix}g_{o,1} & 0\end{bmatrix}}_{\mathbf{c}^T}\underbrace{\begin{bmatrix}x_1\\ x_2\end{bmatrix}}_{\mathbf{x}} + \underbrace{\alpha\begin{bmatrix}g_{o,1} & 0\end{bmatrix}}_{\alpha\,\mathbf{c}^T}\underbrace{\begin{bmatrix}x_1^3\\ x_2^3\end{bmatrix}}_{\mathbf{x}^{\bullet 3}}$$

where Hadamard's product $\mathbf{a} \bullet \mathbf{b} = (a_1b_1, a_2b_2, \ldots, a_kb_k)^T$ and power $\mathbf{a}^{\bullet\rho} = (a_1{}^\rho, a_2{}^\rho, \ldots, a_k{}^\rho)^T$ were used.

Note that in many $G_m - C$ filters the output is the voltage across a capacitor. If x_2, for example, was the output signal in Figure 2 then there would be no output transconductor stage and the output equation would be $y = x_2$, *which is linear.*

The state-space model for the general n^{th} order $G_m - C$ filter with weak 3^{rd} order nonlinearity is

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \alpha\mathbf{A}\mathbf{x}^{\bullet 3}(t) + \mathbf{b}u(t) + \alpha\mathbf{b}u^3(t) \tag{2}$$

$$y(t) = \mathbf{c}^T\mathbf{x}(t) + \alpha\mathbf{c}^T\mathbf{x}^{\bullet 3} \tag{3}$$

where $\mathbf{A} \in \Re^{n\times n}$, $\mathbf{x}, \mathbf{b}, \mathbf{c} \in \Re^{n\times 1}$ and $u(t), y(t) \in \Re$. Again, '$\bullet$' stands for Hadamard product or power. Finally, the input u considered in this work is the standard two-beat signal used in IMD estimation

$$u(t) = k_1 sin\,(w_1 t) + k_2 sin\,(w_2 t)\,. \tag{4}$$

4. FILTER'S CASCADE STRUCTURAL DECOMPOSITION

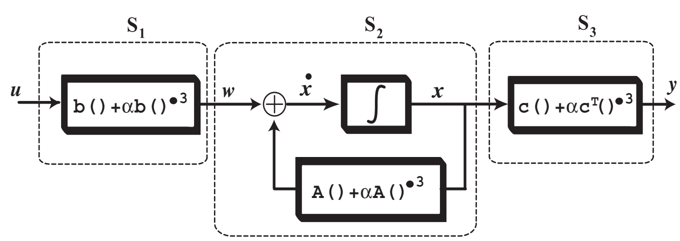

Figure 3: Block diagram of the weakly nonlinear $G_m - C$ filter

The state-space representation of the weakly nonlinear $G_m - C$ filter, Eqs. (2) and (3), is shown in the block diagram of Figure 3. The filter is a cascade of three stages; the input stage, represented by the signal operator S_1, the filter core stage - operator S_2 and the output stage - operator S_3. It is

$$\mathbf{w} = S_1(u), \;\; \mathbf{x} = S_2(\mathbf{w}) \;\text{ and }\; y = S_3(\mathbf{x})$$

The total response of the system is $y = (S_3 \circ S_2 \circ S_1)(u)$.

Two clarifications are in order regarding S_1, S_2 and S_3. 1) Since we are interested in IMD estimation and we use the two-beats input signal (4), only the steady state behavior of the filter (and therefore of each of the stages) is taken into account. To this end, w, $\mathbf{x}$ and y are the steady state responses of the stages[1], and operators S_1, S_2 and S_3 are the corresponding mappings between them (and input u). 2) The input and output stages, i.e. S_1 and S_3 are static functions. Distortion due to static nonlinearities has been studied extensively. The filter core however, S_2, has dynamics making the IMD estimation challenging. To deal with operator S_2 regular perturbation theory was employed [22] and the mathematical details can be found in [20].

Each of the three stages[2] is naturally decomposed into two parts, the linear one (which is the ideal and desirable one) and the nonlinear one (which represents the nonlinearities in the stage). We write:

$$S_i = S_i^{\ell} + S_i^{n}\,, \quad i = 1, 2, 3. \tag{5}$$

The decomposition of the input, S_1, and output, S_3, stages is implied directly from Figure 3.

$$\begin{array}{ll} S_1^{\ell}(u) \triangleq \mathbf{b}\, u & S_1^{n}(u) \triangleq \alpha\, \mathbf{b}\, u^3 \\ S_3^{\ell}(\mathbf{x}) \triangleq \mathbf{c}^T \mathbf{x} & S_3^{n}(\mathbf{x}) \triangleq \alpha\, \mathbf{c}^T \mathbf{x}^{\bullet 3} \end{array} \tag{6}$$

The linear part, S_2^{ℓ}, of S_2 is the steady state response of the (asymptotically stable) linear system $\dot{\mathbf{x}}_{\mathbf{0}} = \mathbf{A}\mathbf{x}_{\mathbf{0}} + \mathbf{w}$, i.e. $S_2^{\ell}(\mathbf{w}) = \mathbf{x}_{\mathbf{0}}$ (steady state). Operator S_2^n is defined by $S_2^n \triangleq S_2 - S_2^{\ell}$ i.e. $S_2^n(\mathbf{w}) = \mathbf{x}_{\mathbf{d}} = \mathbf{x} - \mathbf{x}_{\mathbf{0}}$ is the difference between the steady state responses of the nonlinear and the linear systems shown in Figure 4.

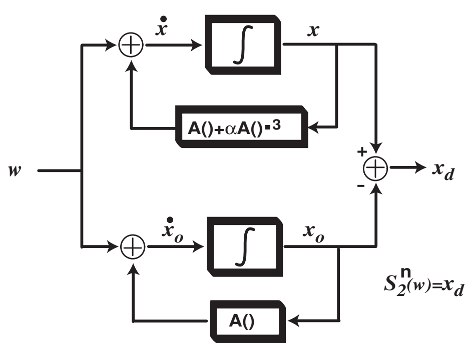

Figure 4: Definition of operator S_2^n

The decomposition of the stages, Eqs. (5), transforms system's block diagram in Figure 3 into that in Figure 5. The cascade of parallel pairs in Figure 5 is equivalent to eight

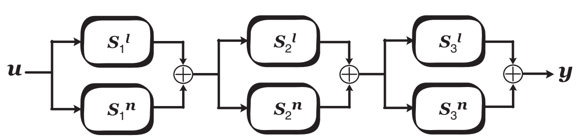

Figure 5: The filter as a cascade of decomposed stages

parallel signals paths from the input u to the output y, each involving either a linear or a nonlinear operator from every stage, namely: $y = (S_3 \circ S_2 \circ S_1)(u) = \big(S_3^{\ell} \circ S_2^{\ell} \circ S_1^{\ell}\big)(u) + \big(S_3^{\ell} \circ S_2^{\ell} \circ S_1^{n}\big)(u) + \big(S_3^{\ell} \circ S_2^{n} \circ S_1^{\ell}\big)(u) + \big(S_3^{n} \circ S_{\eta}^{\ell} \circ S_1^{\ell}\big)(u) + \big(S_3^{\ell} \circ S_2^{n} \circ S_1^{n}\big)(u) + \big(S_3^{n} \circ S_2^{\ell} \circ S_1^{n}\big)(u) + \big(S_3^{n} \circ S_2^{n} \circ S_1^{\ell}\big)(u) + \big(S_3^{n} \circ S_2^{n} \circ S_1^{n}\big)(u)$

[1]The (linear) filter is assumed asymptotically stable by design (i.e. all eigenvalues of matrix A have negative real parts).

[2]Each stage is identified with its operator and the terms *stage* and *operator* are used indistinguishably.

Since we consider weakly nonlinear filters, it is expected that operators S_1^n, S_2^n and S_3^n have a minor influence on the signal compared to that of the corresponding linear ones. Moreover, *compositions* of two or more nonlinear operators S_1^n, S_2^n and S_3^n in a signal path should result in negligible signal components. Therefore, keeping only the signals paths that are linear or include *only* one nonlinear operator results in a good approximation of filter's behavior, i.e.

$$y \approxeq \Big(S_3^{\ell} \circ S_2^{\ell} \circ S_1^{\ell} + S_3^{\ell} \circ S_2^{\ell} \circ S_1^{n} + S_3^{\ell} \circ S_2^{n} \circ S_1^{\ell} + S_3^{n} \circ S_2^{\ell} \circ S_1^{\ell}\Big)(u)$$

The derivation of $(S_3^{\ell} \circ S_2^{\ell} \circ S_1^{\ell})(u)$, $(S_3^{\ell} \circ S_2^{\ell} \circ S_1^{n})(u)$, $(S_3^{\ell} \circ S_2^{n} \circ S_1^{\ell})(u)$ and $(S_3^{n} \circ S_2^{\ell} \circ S_1^{\ell})(u)$ leads to the rapid IMD estimation method for $G_m - C$ filters of any order and any topology that is summarized in the next section.

The derivation of the term $(S_3^{\ell} \circ S_2^{n} \circ S_1^{\ell})(u)$ is challenging since operator S_2^n is realized by the nonlinear dynamical system of Figure 4. To this end regular perturbation techniques were used. More details of this methodology can be found in [20].

5. STEPS TO DERIVE IM_3

The results of this work are summarized in the following procedure for estimating the *third order intermodulation distortion referred to input signal at* w_1, formally defined in (7) where $\mathcal{A}_{2w_2-w_1}$ and $\mathcal{A}_{w_1}$ are the amplitudes of the output signal's (y) components at frequencies $2w_2 - w_1$ and w_1 respectively [6].

$$IM_3 \triangleq \mathcal{A}_{2w_1-w_2}/\mathcal{A}_{w_1} \tag{7}$$

1. *Derive the state-space parameters of the $G_m - C$ filter* : Find matrices $\mathbf{A}, \mathbf{b}$ and $\mathbf{c}$. Derive nonlinearity parameter α of the transconductor analytically or by fitting a third-order polynomial to its $I - V$ characteristic.

2. *Form input signal u* : Choose amplitudes k_1, k_2 and frequencies w_1, w_2 of the input signal $u = k_1 \sin(w_1 t) + k_2 \sin(w_2 t)$.

3. *Calculate $\tilde{\mathbf{h}}_{\mathbf{1}}$, $\tilde{\mathbf{p}}_{\mathbf{1}}$, $\tilde{\mathbf{h}}_{\mathbf{2}}$ and $\tilde{\mathbf{p}}_{\mathbf{2}}$.*

$$\begin{array}{ll} \tilde{\mathbf{h}}_{\mathbf{1}} = -\left(w_1^2\mathbf{I} + \mathbf{A}^2\right)^{-1}\mathbf{A}\mathbf{b}\,, & \tilde{\mathbf{p}}_{\mathbf{1}} = -\left(w_1^2\mathbf{I} + \mathbf{A}^2\right)^{-1} w_1\mathbf{b} \\ \tilde{\mathbf{h}}_{\mathbf{2}} = -\left(w_2^2\mathbf{I} + \mathbf{A}^2\right)^{-1}\mathbf{A}\mathbf{b}\,, & \tilde{\mathbf{p}}_{\mathbf{2}} = -\left(w_2^2\mathbf{I} + \mathbf{A}^2\right)^{-1} w_2\mathbf{b} \end{array}$$

4. *Calculate $\tilde{\mathbf{s}}_{\mathbf{2},-\mathbf{1}}$ and $\tilde{\mathbf{c}}_{\mathbf{2},-\mathbf{1}}$.*

$$\begin{array}{lcl} \tilde{\mathbf{s}}_{\mathbf{2},-\mathbf{1}} & = & \tilde{\mathbf{h}}_{\mathbf{1}}^{\bullet\,2} \bullet \tilde{\mathbf{h}}_{\mathbf{2}} + 2\tilde{\mathbf{h}}_{\mathbf{1}} \bullet \tilde{\mathbf{p}}_{\mathbf{1}} \bullet \tilde{\mathbf{p}}_{\mathbf{2}} - \tilde{\mathbf{p}}_{\mathbf{1}}^{\bullet\,2} \bullet \tilde{\mathbf{h}}_{\mathbf{2}} \\ \tilde{\mathbf{c}}_{\mathbf{2},-\mathbf{1}} & = & \tilde{\mathbf{p}}_{\mathbf{1}}^{\bullet\,2} \bullet \tilde{\mathbf{p}}_{\mathbf{2}} + 2\tilde{\mathbf{h}}_{\mathbf{1}} \bullet \tilde{\mathbf{p}}_{\mathbf{1}} \bullet \tilde{\mathbf{h}}_{\mathbf{2}} - \tilde{\mathbf{h}}_{\mathbf{1}}^{\bullet\,2} \bullet \tilde{\mathbf{p}}_{\mathbf{2}} \end{array}$$

5. *Calculate* $\mathbf{F} \triangleq \left((2w_1 - w_2)^2\mathbf{I} + \mathbf{A}^2\right)^{-1}$, $S_{2,-1}$ *and* $C_{2,-1}$.
Remark: Terms corresponding to linear or non-existing stages of the filter must be removed from $S_{2,-1}$ and

$C_{2,-1}$.

$$S_{2,-1} = \underbrace{-\mathbf{c}^T\mathbf{F}\mathbf{A}\mathbf{b}}_{input} \underbrace{-\mathbf{c}^T\mathbf{F}\left(\mathbf{A}^2\tilde{\mathbf{s}}_{\mathbf{2},-\mathbf{1}} - (2w_1 - w_2)\mathbf{A}\tilde{\mathbf{c}}_{\mathbf{2},-\mathbf{1}}\right)}_{filter\ core} \underbrace{+\mathbf{c}^T\tilde{\mathbf{s}}_{\mathbf{2},-\mathbf{1}}}_{output}$$

$$C_{2,-1} = \underbrace{-(2w_1 - w_2)\mathbf{c}^T\mathbf{F}\mathbf{b}}_{input} - \underbrace{-\mathbf{c}^T\mathbf{F}\left(\mathbf{A}^2\tilde{\mathbf{c}}_{\mathbf{2},-\mathbf{1}} + (2w_1 - w_2)\mathbf{A}\tilde{\mathbf{s}}_{\mathbf{2},-\mathbf{1}}\right)}_{filter\ core} \underbrace{+\mathbf{c}^T\tilde{\mathbf{c}}_{\mathbf{2},-\mathbf{1}}}_{output}$$

6. *Calculate* :

$$\mathcal{J}(w_1, w_2) = \frac{\sqrt{(S_{2,-1})^2 + (C_{2,-1})^2}}{\left|\mathbf{c}^T\left(jw_1\mathbf{I} - \mathbf{A}\right)^{-1}\mathbf{b}\right|}$$

7. *Calculate IM_3 relatively to frequency component at w_1.*

$$IM_3 \approx |\alpha|\frac{3\,k_1k_2}{4}\,\mathcal{J}(w_1, w_2) \qquad (8)$$

Note that function $\mathcal{J}(w_1, w_2)$ depends only on the model parameters of the ideal filter, i.e. matrices $\mathbf{A}, \mathbf{b}$ and $\mathbf{c}$, and on frequencies w_1, w_2; *it is independent of the amplitudes* k_1, k_2.

It is worth comparing expression (8) to that of IM_3 introduced by a *static* weakly nonlinear function of the form $v = f(u) = a_1u + a_2u^2 + a_3u^3 + \ldots$ where u is given by (4). In this case IM_3 referred to input signal at w_1, is $IM_3^{static} = \left|\frac{a_3}{a_1}\right|\frac{3k_1k_2}{4}$, e.g. [5], [6]. The ratio a_3/a_1 corresponds to nonlinearity parameter "α" in the model of transconductors given by (1). Moreover, it can be shown that when the filter has no dynamics, or, when $w_1, w_2 \to 0$ while w_1/w_2 remains fixed, then $\mathcal{J}(w_1, w_2) \to 1$. *Therefore, in the lack of dynamics, or in relatively very low frequencies, IM_3 derived using the proposed method equals IM_3^{static}.*

The graph of $\mathcal{J}(w_1, w_2)$ is shown in Figure 6 for the Tow-Thomas bandpass biquad $G_m - C$ (centered at $10.7Mhz$) discussed in the simulation Section 6. The maximum appears near $(10.7MHz, 10.7MHz)$.

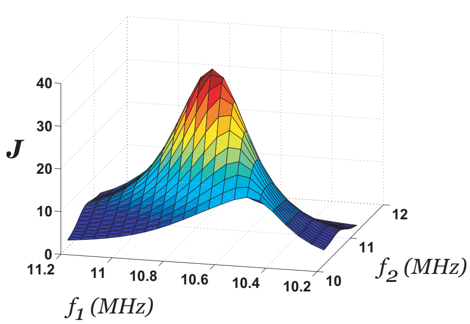

Figure 6: $J(w_1, w_2)$ in linear scale

6. SPICE AND MATLAB SIMULATION

A $G_m{-}C$ Tow-Thomas bandpass biquad was designed in a standard $0.5\mu m$ CMOS process using Cadence and was simulated in SpectreS (SPICE) using bsim3v3 transistor models. Also, the circuit was modeled and simulated in Simulink (MATLAB).

Amplitudes		f_1	f_2	$2f_1 - f_2$
	MHz	10.6	10.5	10.7
k_1=100mV, k_2=100mV	Simulink(dB)	-20.41	-21.69	-58.19
	Theory(dB)	-20.50	-21.85	-58.59
	Cadence(dB)	-19.57	-20.97	-56.82
	Error(dB)	0.93	0.88	1.77
k_1=10mV, k_2=10mV	Simulink(dB)	-40.50	-41.85	-118.60
	Theory(dB)	-40.50	-41.85	-118.59
	Cadence(dB)	-39.64	-41.14	-116.20
	Error(dB)	0.86	0.71	2.39
k_1=1mV, k_2=1mV	Simulink(dB)	-60.50	-61.85	-178.60
	Theory(dB)	-60.50	-61.85	-178.60
	Cadence(dB)	-59.64	-61.14	-176.43
	Error(dB)	0.86	0.71	2.17

Table 1: Simulation and theoretical results.

The schematic of the band-pass filter is as that in Figure 2, but without the output stage, the output is the voltage of the second capacitor, $y = x_2$. *Since there is no output stage, the calculation of $S_{2,-1}$ and $C_{2,-1}$ is done by ignoring the last (output) term in their expressions.* The center frequency of the bandpass filter is $f_o = 10.7Mhz$ and the quality factor is $Q = 20$. The fully differential transconductor used in the filter is shown in Figure 7.

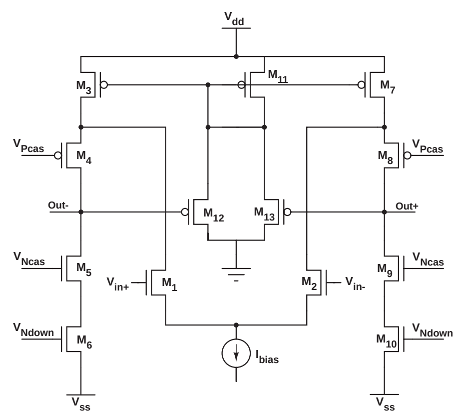

Figure 7: The transconductor's circuit

The values of the transconductances and capacitors of the filter are $g_{1,I} = 31.26\mu A/V$, $g_{1,1} = -31.26\mu A/V$, $g_{2,1} = 625.2\mu A/V$, $g_{1,2} = -625.2\mu A/V$ and $C_1 = C_2 = 9.3054pF$. Parameter α was estimated by curve fitting a third order polynomial to the I-V characteristic of the transconductor, and $\alpha = -0.0535V^{-2}$.

The test input signal used for simulation and theoretical estimation is given by (4). Different combinations of amplitude pairs (k_1, k_2) and frequency pairs (f_1, f_2), around the centrer frequency of $10.7MHz$ were used.

The amplitudes of the frequency components at the output, at frequencies f_1, f_2 and $2f_1 - f_2$ are shown in Tables 1 and 2, all in logarithmic scale: $20\log_{10}$(Amplitude in Volts). The *error* values are the differences between SpectreS and theory.

In all cases the theoretical results were very close to those of SpectreS simulation. The largest errors appeared with the smallest amplitudes, in which cases, the intermodulation signal at $2f_1 - f_2$ is about $110dB$ below the referenced input at w_1 and therefore negligible for most applications.

Amplitudes		f_1	f_2	$2f_1 - f_2$
	MHz	10.7	10.8	10.6
k_1=10mV k_2=100mV	Simulink	-40.01	-20.68	-97.23
	Theory(dB)	-40.01	-20.64	-97.20
	Cadence(dB)	-39.21	-20.13	-96.52
	Error(dB)	0.80	0.51	0.68
k_1=10mV k_2=10mV	Simulink	-40.00	-40.64	-117.12
	Theory(dB)	-40.00	-40.64	-117.20
	Cadence(dB)	-39.20	-40.11	-114.97
	Error(dB)	0.80	0.53	2.23
k_1=10mV k_2=1mV	Simulink	-40.00	-60.64	-137.12
	Theory(dB)	-40.00	-60.64	-137.20
	Cadence(dB)	-39.20	-60.11	-134.94
	Error(dB)	0.80	0.53	2.26

Table 2: Simulation and theoretical results.

7. CONCLUSIONS

A fast algorithm to estimate the intermodulation distortion of band-pass $G_m - C$ filters with fully differential weakly nonlinear transconductors has been introduced. It is based on state-space modeling and is independent of the order or the particular topology of the $G_m - C$ filter. The algorithm has been verified using a band-pass $G_m - C$ biquad with fully differential weakly nonlinear transconductors designed in a 0.5μm standard CMOS process. The theoretical results were found in good agreement with simulation.

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