A Fast State-Space Algorithm to Estimate Harmonic Distortion in Fully Differential Weakly Nonlinear $G_m - C$ Filters

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***Abstract*— In this paper, we present a fast algorithm to derive the harmonic distortion in fully balanced $G_m - C$ filters. It is based on state-space modeling and decomposition of the filter into a cascade of an input stage, a core stage and an output stage. This approach results in compact expressions for the distortion that involve explicitly the structural matrices of the filter and the values of the circuit elements. The algorithm was verified using a third-order low pass Butterworth filter designed in a 0.5μm CMOS process. The theoretical results are found to be in good agreement with CADENCE and MATLAB simulations.**

I. Introduction

In the past two decades, a great interest has emerged in designing continuous time filters using the transconductance-capacitor ($G_m - C$) technique. As an alternative to MOSFET-C techniques, $G_m - C$ filters have been widely employed in high frequency applications, such as telecommunications, digital video, and intermediate frequency (IF) filters, due to their tunability, low power, and wide operational bandwidth [1]-[10]. However, to achieve high frequency performance, linearity is usually sacrificed in the choice of (simple) transconductor blocks. This results in non-negligible distortion at the transconductors and therefore at the $G_m - C$ filters' output.

When a sinusoidal signal of frequency ω is applied to the input of the filter, the steady state response of the output signal consists of not only the fundamental frequency component ω, but also those at harmonics of the input waveform, *i.e.*, $2\omega, 3\omega, 4\omega, \cdots$. These higher order terms form the *harmonic distortion*. The total harmonic distortion (THD) is defined as

$$\text{THD} \;=\; 20\lg\sqrt{\frac{V_{h2}^2 + V_{h3}^3 + V_{h4}^2 + \cdots}{V_f^2}} \quad \text{(in dB)}$$

where V_f is the amplitude of the fundamental frequency component and V_{hi} is the amplitude of the i^{th} harmonic component.

Harmonic distortion is a major issue in anti-aliasing filters in analog-to-digital converters (ADC). Suppose, for example, the input signal to the ADC is a sinusoidal at 400kHz and the sampling frequency is 1.5MHz. The third order harmonic component at the output of the anti-aliasing filter will be at 1200kHz and thus aliased at 300kHz. This undesirable component at 300kHz directly degrades the actual SNR behavior of the analog-to-digital conversion process by corrupting the useful signal.

In addition, harmonic distortion as well as noise dictate the dynamic range of a filter [11]. Over the past decades, significant effort has been made to optimize filters in terms of their dynamic range and power dissipation. Companding and scaling have been employed to improve the dynamic range, and many circuit techniques have been developed to reduce nonlinearity [1], [12]-[17]. Precise knowledge of harmonic distortion of the filters has been of important guidance for the above efforts.

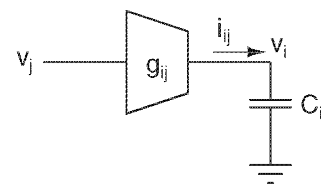

Fig. 1. The basic $G_m - C$ block.

Techniques have been developed to calculate the harmonic distortion of a filter. The Volterra series approach is used in almost all of them [18]-[20] where the nonlinear system is decomposed into an infinite number of subsystems with polynomial nonlinearities and the harmonic components are evaluated separately for each subsystem.

Another approach was presented in [12] where partial transfer functions from the input to internal nodes of the filter were used to derive the output harmonic distortion components.

Some of these approaches, e.g. [20], provide some insight into the dependence of the distortion on component values. However, in most cases of higher order filters, especially those without a special topology, deriving closed-form analytical results using a Volterra series or partial-transfer-function based approach is highly complicated, if possible at all, in most cases.

In contrast to the widely followed path, this paper proposes a fast harmonic distortion estimation algorithm that is based on state-space representation of the filter. This approach leads to simple analytic formulas that apply directly to *Gm-C* filters of *any order* and any fully balanced topology. The algorithm has been verified using SPICE (CADENCE) and Simulink simulation. Theory and simulation results are found in good agreement.

The paper is organized as follows. Section II introduces the state space modeling of fully differential weakly nonlinear $G_m - C$ filters. Section III, presents the fast algorithm for harmonic distortion estimation. Section IV presents the results of the algoritm and compares them to SPICE (CADENCE) simulation results.

II. Fully Differential Weakly Nonlinear $G_m - C$ Filters and their State Space Representation

The basic building block of a $G_m - C$ filter consists of a transconductor and a capacitor as shown in Figure 1.

The transconductor is connected between nodes j (input) and i (output). Its gain is $g_{i,j}$ and its output current $i_{i,j}$ charges capacitor C_i. An ideal transconductor implies a linear relation between input voltage and output current

$$i_{i,j} = g_{i,j} v_j. \tag{1}$$

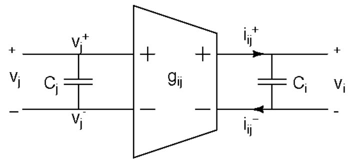

Fig. 2. Fully differential $G_m - C$ block.

In reality, however, the input-output relation is not purely linear but it contains higher order nonlinear terms as well.

In most $G_m - C$ filter designs, fully differential (fully balanced) transconductors are preferred, in this case the basic $G_m - C$ block is shown in Figure 2.

In the rest of this paper we consider filters with fully differential transconductors. For notational simplicity though, the simple single ended diagram is used.

Because of their balanced structure, fully differential transconductors exhibit significantly more linear behavior than the single ended ones. Balanced structures tend to eliminate, or at least reduce significantly, *even order* distortion terms resulting in an output current of the form

$$i_{i,j} = g_{i,j}v_j + g_{i,j}^{(3)}{v_j}^3 + g_{i,j}^{(5)}{v_j}^5 + \cdots$$

Moreover, in most practical cases, the fifth and higher order terms are negligible compared to the third order term and can be safely ignored, i.e.,

$$i_{i,j} = g_{i,j}v_j + g_{i,j}^{(3)}{v_j}^3. \tag{2}$$

These assumptions are typical in analyzing the distortion of the filters with fully balanced transconductors [11][12][19], and are adopted here as well. Also, in many practical cases, the coefficient of the third order power, $g_{i,j}^{(3)}$, is proportional to $g_{i,j}$, *i.e.*, $g_{i,j}^{(3)} = \varepsilon g_{i,j}$. The (small) constant ε can be easily extracted analytically or numerically.

$$i_{i,j} = g_{i,j}v_j + \varepsilon g_{i,j}v_j^3. \tag{3}$$

A (ideally) linear $G_m - C$ filter can be treated as a general single-input, single-output n^{th} order dynamical system with input u, output y and state vector $\mathbf{v} = (v_1, v_2, ..., v_n)^T$ where v_i is the voltage of the i^{th} capacitor $i = 1, 2, \ldots$. In this case, the state space equations of the filter are

$$\begin{array}{rcl}\dot{\mathbf{v}} & = & \mathbf{A}\mathbf{v} + \mathbf{B}u \\ y & = & \mathbf{C}\mathbf{v}\end{array} \tag{4}$$

where $\mathbf{A} = [g_{i,j}/C_i]_{i,j=1}^n$ is the system matrix, $\mathbf{B} = (g_i/C_i)_{i=1}^n$ is the input vector and g_i is the transconductor from the input to the i^{th} capacitor. Finally $\mathbf{C} = {(c_i)_{i=1}^n}^T$ is the output row vector. Note than in many filter designs the output is simply a state variable of the filter, i.e. $\mathbf{C} = (0, \ldots, 0, 1, 0, \ldots, 0)$.

When the transconductors exhibit weak nonlinearity modeled by equation (3), system (4) is replaced by

$$\begin{array}{rcl}\dot{\mathbf{v}} & = & \mathbf{A}\mathbf{v} + \mathbf{B}u + \varepsilon\mathbf{A}\mathbf{v}^{\bullet(3)} + \varepsilon\mathbf{B}u^3 \\ y & = & \mathbf{C}\mathbf{v} + \varepsilon\mathbf{C}\mathbf{v}^{\bullet(3)}\end{array} \tag{5}$$

where $\mathbf{v}^{\bullet(3)} = (v_1^3, v_2^3, ..., v_n^3)^T$ is the third entry-wise power of $\mathbf{v}$. System (5) is used as our filter model in this work [1].

[1]In the case where the assumption $g_{j,i}^{(3)} = \varepsilon g_{j,i}$ is not satisfied, Equations (5) can be replaced by (6) below where $\mathbf{E}$, $\mathbf{F}$, and $\mathbf{K}$ are the corresponding matrices for the 3rd order terms.

$$\begin{array}{rcl}\dot{\mathbf{v}} & = & \mathbf{A}\mathbf{v} + \mathbf{E}\mathbf{v}^{\bullet(3)} + \mathbf{B}u + \mathbf{F}u^3 \\ y & = & \mathbf{C}\mathbf{v} + \mathbf{K}\mathbf{v}^{\bullet(3)},\end{array} \tag{6}$$

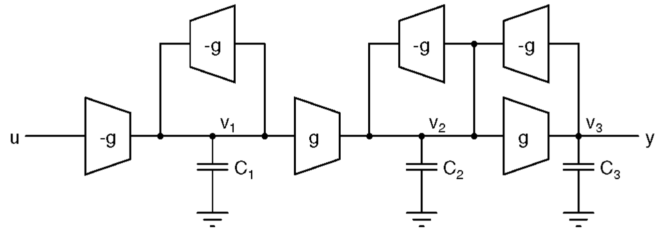

Fig. 3. A third order Butterworth low pass $G_m - C$ filter.

Although it is desirable that parameter ε be small, note that the relative contribution of the nonlinearities in (5) is determined not by ε itself but by the magnitudes of the products $\varepsilon\|\mathbf{v}\|_2^2$ and εu^2. Moreover, the linear transformation $\mathbf{v} = \eta\mathbf{z}$, $u = \eta w$ and $y = \eta q$ results to $\dot{\mathbf{z}} = \mathbf{A}\mathbf{z} + \mathbf{B}w + \varepsilon\eta^2\mathbf{A}\mathbf{z}^{\bullet(3)} + \varepsilon\eta^2\mathbf{B}w^3$ and $q = \mathbf{C}\mathbf{z} + \varepsilon\eta^2\mathbf{C}\mathbf{z}^{\bullet(3)}$ where ε has been "replaced" by $\varepsilon\eta^2$.

Example: The above definitions are illustrated using the third order Butterworth low pass filter in Figure 3. With $C_1 = C_2 = C_3 = 8pF$ and $g = 53.8\mu A/V$ its transfer function is

$$H(s) = \frac{Y(s)}{U(s)} = \frac{1}{\left(\frac{s}{\omega_0} + 1\right)\left((\frac{s}{\omega_0})^2 + (\frac{s}{\omega_0}) + 1\right)} \tag{7}$$

where $\omega_0 = g/C = 2\pi f_0$ and the $-3dB$ frequency is $f_0 = 1.07\ Mhz$. The voltage vector is $\mathbf{v} = (v_1, v_2, v_3)^T$ and its entry-wise (Hadamard) third order power is $\mathbf{v}^{\bullet(3)} = (v_1^3, v_2^3, v_3^3)^T$. The matrices $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ are

$$\mathbf{A} = \begin{bmatrix} -\frac{g}{C_1} & 0 & 0 \\ \frac{g}{C_2} & -\frac{g}{C_2} & -\frac{g}{C_2} \\ 0 & \frac{g}{C_3} & 0 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} -\frac{g_1}{C_1} \\ 0 \\ 0 \end{bmatrix}, \mathbf{C} = (0, 0, 1). \tag{8}$$

Assuming that fully differential weakly nonlinear transconductors are used, then following equations (5), the state space equations of this filter are

$$\begin{array}{rcl}\dot{\mathbf{v}} & = & \mathbf{A}\mathbf{v} + \mathbf{B}u + \varepsilon\mathbf{A}\mathbf{v}^{\bullet(3)} + \varepsilon\mathbf{B}u^3 \\ y & = & \mathbf{C}\mathbf{v}\end{array} \tag{9}$$

Note that there is no nonlinear term in the output equation since the output is identical to a state variable (the voltage across a capacitor).

III. Estimation of Harmonic Distortion

A. Total Harmonic Distortion

Consider the general n^{th} order fully differential weakly nonlinear $G_m - C$ filter and its mathematical model given by (5). It is fair to assume the filter is asymptotically stable, i.e., the eigenvalues of matrix $\mathbf{A}$ have negative real part.

The following theorem summarizes the results of this work and provides a simple and fast algorithm for deriving the harmonic distortion. It gives a good approximation of the output signal of fully differential weakly nonlinear $G_m - C$ filters driven by a sinusoidal input. The approximate output is derived by decomposing the filter into a cascade of three stages, namely, the input stage, the filter core and the output stage, and calculating the distortion that each one contributes to the output. The challenging part is to solve the system of weakly nonlinear differential equations modeling the core of the filter. This is done approximately using regular perturbation theory.

Theorem [21] : *The output, y, of a fully differential weakly nonlinear $G_m - C$ filter, modeled by equations (5) and driven by the sinusoidal input signal $u = a\sin(\omega t)$, is*

$$y(t) \approxeq S_c \cos(\omega t) + S_s \sin(\omega t) + D_c \cos(3\omega t) + D_s \sin(3\omega t)$$

where the constants S_c, S_s, D_c and D_c are given by

$$\begin{aligned} S_c &= -a\omega\mathbf{C}(\omega^2\mathbf{I} + \mathbf{A}^2)^{-1}\mathbf{B} \\ S_s &= -a\mathbf{C}(\omega^2\mathbf{I} + \mathbf{A}^2)^{-1}\mathbf{AB} \end{aligned}$$

$$\begin{aligned} D_c = \tfrac{3}{4}a^3\varepsilon\mathbf{C}(9\omega^2\mathbf{I} + \mathbf{A}^2)^{-1}\omega\mathbf{B} & \quad \leftarrow \textit{input} \\ -\varepsilon\mathbf{C}(9\omega^2\mathbf{I} + \mathbf{A}^2)^{-1}\mathbf{A}(\mathbf{A}\mathbf{p_3} + 3\omega\mathbf{q_3}) & \quad \leftarrow \textit{core} \\ +\varepsilon\mathbf{C}\mathbf{p_3} & \quad \leftarrow \textit{output} \end{aligned} \tag{10}$$

$$\begin{aligned} D_s = \tfrac{1}{4}a^3\varepsilon\mathbf{C}(9\omega^2\mathbf{I} + \mathbf{A}^2)^{-1}\mathbf{AB} & \quad \leftarrow \textit{input} \\ -\varepsilon\mathbf{C}(9\omega^2\mathbf{I} + \mathbf{A}^2)^{-1}\mathbf{A}(\mathbf{A}\mathbf{q_3} - 3\omega\mathbf{p_3}) & \quad \leftarrow \textit{core} \\ +\varepsilon\mathbf{C}\mathbf{q_3} & \quad \leftarrow \textit{output} \end{aligned} \tag{11}$$

and the constants $\mathbf{p_3}$, $\mathbf{q_3}$ are given in terms of $\mathbf{r}$, $\mathbf{s}$

$$\begin{aligned} \mathbf{p_3} &= \frac{1}{4}\mathbf{r}^{\bullet(3)} - \frac{3}{4}\mathbf{r} \bullet \mathbf{s}^{\bullet(2)} \\ \mathbf{q_3} &= -\frac{1}{4}\mathbf{s}^{\bullet(3)} + \frac{3}{4}\mathbf{r}^{\bullet(2)} \bullet \mathbf{s} \end{aligned}$$

where

$$\begin{aligned} \mathbf{r} &= -a(\omega^2\mathbf{I} + \mathbf{A}^2)^{-1}\omega\mathbf{B} \\ \mathbf{s} &= -a(\omega^2\mathbf{I} + \mathbf{A}^2)^{-1}\mathbf{AB} \end{aligned}$$

For any two n-vectors it is $\mathbf{x} \bullet \mathbf{y} = (x_1y_1, x_2y_2, \ldots, x_ny_n)$. □

In most cases, the third order harmonic distortion component is dominant, with the possible exception of certain steep high pass filters, and the higher order harmonic components can be ignored. Therefore the *total harmonic distortion* can be well approximated by the third order one. This is further supported by simulation results in Figure 7 as well.

From the theorem we have that the third order distortion and therefore an approximation of the Total Harmonic Distortion of the filter is given by the expression

$$\text{THD} \approx 10\log_{10}\left(\frac{D_c^2 + D_s^2}{S_c^2 + S_s^2}\right), \quad (dB) \tag{12}$$

Note that D_c, D_s are proportional to εa^3 and S_c, S_s proportional to a, where ε is the nonlinearity parameter of the transconductor, equation (6), and a is the input amplitude. Therefore it is

$$\text{THD} \approx 20\log_{10}(|\varepsilon|) + 40\log_{10}(a) + \varphi(\mathbf{A}, \mathbf{B}, \mathbf{C}, \omega)$$

where φ is a function of the system's parameters and the frequency ω *only*. If $|\varepsilon|$ is reduced to $|\varepsilon|/10$, the power of the third order harmonic distortion and the THD will both drop approximately by $20dB$. If a is reduced to $a/10$, the power of the harmonic distortion and the THD will drop by $60dB$ and $40dB$ respectively.

Remark: The three terms on the right hand side of (10) and (11), ordered from top to bottom, correspond to the distortion introduced by the *input*, *core* and *output* stages of the filter respectively. If any of these stages is linear, or it is missing, then the corresponding terms should be removed. For instance, the filter in Figure 3 does not have output stage since $y = v_3$ and so the terms $\varepsilon\mathbf{C}\mathbf{p_3}$ and $\varepsilon\mathbf{C}\mathbf{q_3}$ must be ignored.

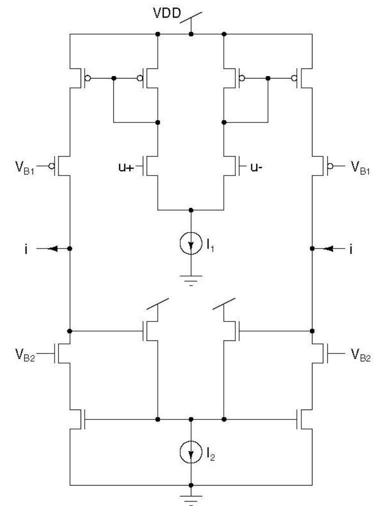

Fig. 4. Schematic of the transconductor used in Cadence simulation.

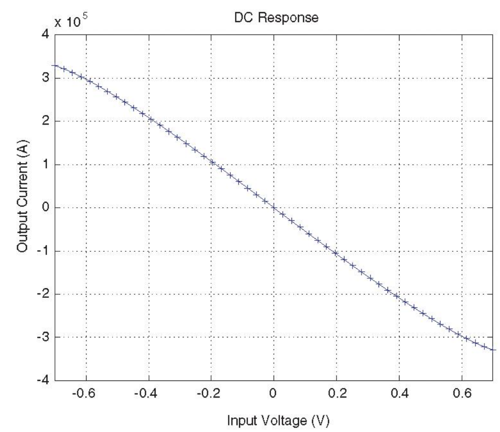

Fig. 5. I-V characteristic of the transconductor in Fig. 4.

IV. Simulations

The proposed THD derivation process was verified using the third order Butterworth low pass filter in Figure 3 and SPICE simulation. The typical, fully balanced, cascode-output transconductor shown in Figure 4 was used. The simulation was done using the 49^{th} level SPICE transistor models and AMI 0.5μm technology file (AMIS C5). SPICE simulations were done for: 1) Extraction of the (linear) transconductance, g, and the ε parameter of the transconductor. To this end, a DC sweep provided the $I - V$ characteristic of the transconductor shown in Figure 5. The data was exported to MATLAB to perform a curve fitting using a third order polynomial. The estimated (linear) transconductance was $g = 53.8\mu A/V$, the second order coefficient was on the order of 10^{-20}, therefore negligible as expected due to the balance structure of the transconductor, and $\varepsilon = -0.229\ V^{-2}$. 2) For extracting the harmonic components at the output of the filter and calculating the THD. To this end, transient simulation was done for every frequency point and the resulting waveform was feed into the FFT function of SPICE.

The values of the THD derived using SPICE (CADENCE) simulation and those derived using the algorithm implemented in MATLAB are shown in Figure 6. Three input amplitudes $a = 0.1V$, $0.2V$ and $0.4V$ were used and the frequency range was from $10kHz$ to $4MHz$.

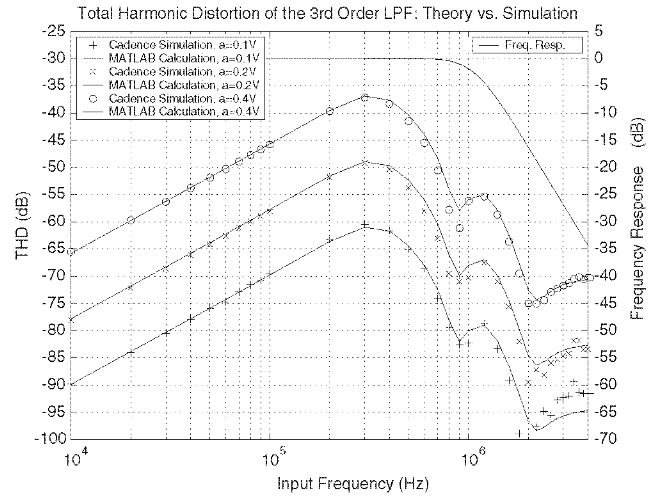

Fig. 6. Simulation results of the low-pass filter shown in Figure 3. The top solid line is the frequency response. The three lower solid lines are the distortions of the filter when driven by $0.1V$, $0.2V$ and $0.4V$ input signal. "o", "x" and "+" correspond to SPICE (CADENCE) measurements.

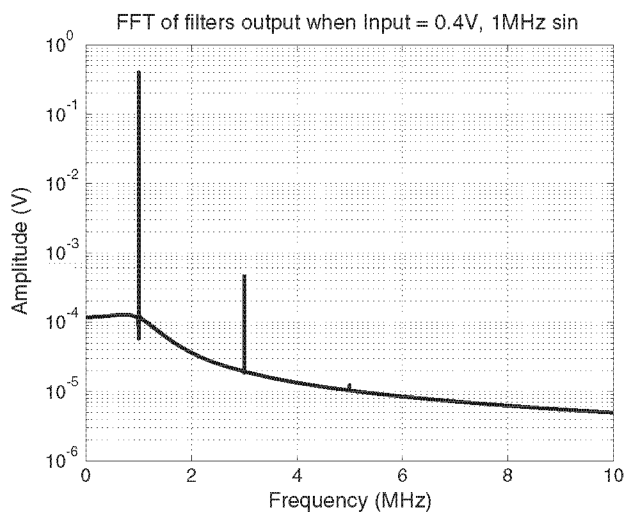

Fig. 7. Fast Fourier transform of the low-pass filter output in Figure 3 driven by input signal $u(t) = 0.4V \sin(2\pi \cdot 10^6 t)$.

The FFT of the output signal is shown in Figure 7. The fundamental and the third harmonic are shown clearly while the fifth is barely there (it's amplitude is about 1/100 that of the third harmonic).

The simulation and theoretical results are in good agreement especially below the $f_0 = g/(2\pi C) = 1.07Mhz$ frequency. A major part of the error is due to bandwidth limitation of the transconductors and the phase shift it introduces.

V. Conclusions

The paper presents a fast algorithm to estimate the THD of $G_m - C$ filters with fully differential, weakly nonlinear transconductors. The algorithm is based on state-space modeling and decomposition of the filters. The algorithm was applied to low pass Butterworth filter and the results were compared with SPICE (CADENCE) simulation and found to be in good agreement. The advantage of the presented algorithm is that all calculations are done explicitly in terms of the structural matrices $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ of the filter and not implicitly through partial or higher order transfer functions as done in previously published techniques. Moreover, the simple analytic formulas given in the theorem in Section III apply directly to *Gm-C* filters of *any order*.

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